FPGA acceleration using High-Level Languages of a Monte-Carlo method for pricing complex options

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Abstract

In this paper we present an FPGA implementation of a Monte-Carlo method for pricing Asian Options using Impulse C and floating-point arithmetic. In an Altera Stratix-V FPGA, a 149x speedup factor was obtained against an OpenMP-based solution in a 4-core Intel Core i7 processor. This speedup is comparable to that reported in the literature using a classic HDL-based methodology, but the development time is significantly reduced. Additionally, the use of a HLL-based methodology allowed us to implement a high-quality gaussian random number generator, which produces more precise results than those obtained with the simple generators usually present in HDL-based designs.

Keywords: High Level Language Synthesis, Field Programmable Gate Arrays, financial data processing, parallel machines, floating-point arithmetic.

1. Introduction

Options are derivative financial instruments which consist on a contract that gives the right to its holder to buy or sell some amount of the underlying

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asset at a future date, for a pre-specified price. Options do not imply any obligation to buy or sell, they just grant the right to execute a transaction at an agreed price. Although contracts similar to options have existed since ancient times, the first time that contemporary financial options were traded in an orderly fashion was in 1973, with the opening of the CBOE in Chicago. Since then, option trading has grown tremendously, and nowadays options are traded on a wide range of assets: stocks, indexes, currencies, commodities, etc. An option giving the right to buy a certain asset is named a “call”, while an option giving the right to sell a certain asset is named a “put”.

Options are useful for hedging the risk. For example, if you know that you are going to buy oil in the future, you can buy call options for protecting yourself against a sudden rise in its price. If the price of oil does not rise, you simply let the option expire, but if there is an unexpected increase in its price, you can exercise the option to buy at a lower price. Alternatively, options can also be used for speculative purposes. If you suspect that there is going to be market crash soon, you can buy put options for a given equity. If the market crash finally occurs, you can buy that equity and then exercise the option to sell at a higher price. The buyer of the option pays an amount named premium to the originator of the option. Depending on the premium paid, it will be profitable or not to exercise the option at its expiration date.

Option valuation is a complex problem. There are many pricing models, which usually depend on the current price of the asset being negotiated, the price at which it will be traded (strike price), the cost of holding the asset (e.g. interests), the time to the expiration of the option, and volatility of the asset price during the life of the option. Depending on the style of the option, pricing may require complex numerical models. For example, in the case of Asian option pricing, an asset price is calculated in terms of the mean value during its lifetime. For American options, the asset price is calculated at its exercise point which, in contrast with the European option style, can be different to the option’s expiration time. This way, Asian options are cheaper (as their potential benefits are lower) and less sensitive to market peaks and manipulation. However, the mean value feature of Asian option pricing has the problem of not having a closed-form solution for the pricing equation. In such cases, some approximation form [1] or simulation method must be applied, typically Monte-Carlo simulations. This papers focuses on the usage of Monte-Carlo simulations due to their ubiquity in the High-Performance Computing world. These Monte-Carlo simulations are based on the computation of random experiments following a determined probabilistic
distribution. However, the low convergence rate of such methods creates the need of running huge number of path simulations in order to converge to the proper value. This is translated into simulations taking enormous amounts of time.

In those cases were pricing is based on complex models, there is a competitive advantage for those traders which can compute the models faster. The sooner that you can realize that the price of an option is favorable, the less possibilities exist that other trader buys the option and profits from it. However, not only the speed in getting the estimated price is relevant, but the accuracy of the estimation is also of capital relevance to decide if the price of an option is favorable. The ubiquity of high-performance computing systems has made difficult to stand out in the financial market: Nowadays anyone can use a powerful computer in order to calculate an asset price before his competitors. It is therefore necessary to find out new technologies which allow traders to significantly accelerate simulations in order to provide a relevant advantage to the competition.

FPGAs have already been proved as good candidates for algorithm acceleration since they allow engineers to design custom architectures optimized to solve a specific problem by taking advantage of much higher degrees of parallelism than CPUs. One of the main drawbacks in hardware development is the use of hardware description languages (HDLs), whose utilization entails high development times and painful verification processes. In order to ease this process, in the recent years we have seen the advent of high level languages (HLL) compilers capable of generating HDL code from high level languages such as C or derivates. In this work, we use the Impulse C tool to generate a hardware description of a highly accurate Monte-Carlo method for pricing Asian Options and compare it against a multicore solution based on the OpenMP programming framework. It is shown that a HLL-based methodology is not only capable of providing similar accelerations to those of a HDL-based design, but also with a better accuracy, since more precise algorithms can be implemented.

The remainder of the paper is organized as follows. In section 2 an overview of the Asian option model is presented, as well as the mathematical mechanisms used in this paper to simulate this model. A keynote on the High-Level Language used during the implementation phase (Impulse C) is pictured in section 3. The implementation procedure is depicted along section 4 and its results are presented at section 5, compared to a multicore CPU solution. Finally, some conclusions are drawn in section 6.
2. Asian options and Monte-Carlo simulations

2.1. Asian options

As it has been set along the introduction of this paper, options are contracts between two parts regarding some asset or stock price. The price in the contract is known as the exercise price or strike price \((E)\), and the date in the contract is known as the expiration date or maturity \((T)\). When acquiring the option, the “owner to be” must pay to the option issuer the fixed price for the option to be created. The calculation of the price usually depends on the desired interest rate \(r\) and the expiration time of the option as shown in Eq.(1).

\[
\text{option value} = e^{-rT} \text{option payoff} \tag{1}
\]

Once the option has been created, the option issuer must accept the decision of the option owner regarding the execution or not of the call/put operation. Under those terms the acquiring price of an option must be carefully calculated, therefore both the accuracy and the latency of this calculation become of critical relevance.

Regarding the time when the option can be exercised, we can distinguish between American or European options. The former ones allow the holder to exercise the option at any time up to the expiration date. However, European options can only be exercised at maturity. Traditional Asian options behave like the European ones regarding the moment they can be exercised at.

The payoff of an option is the quantity used for the option exercise price calculation process. The payoff of a call option is then the difference between the price of the underlying asset at the exercise time \((Price(T))\) and the exercise price for both American and European options. If this difference is lower than zero then the payoff will be zero because the option will never be exercised (see Eq.(2)). Similarly, the payoff of a put American or European option is the difference between the exercise price and the asset price at the exercise time unless this difference is lower than zero (see Eq.(3)).

\[
P_{\text{call}} = \max(Price(T) - E, 0) \tag{2}
\]

\[
P_{\text{put}} = \max(E - Price(T), 0) \tag{3}
\]

Asian options come to remove the risk of market manipulation, since the strike price is computed as the mean price of the underlying asset over the
life of the option [2] as shown in Eq.(4) and Eq.(5).

\[ P_{\text{call}} = \max \left( \frac{1}{n} \sum_{i=1}^{n} Price(t_i) - E, 0 \right) \] \tag{4}

\[ P_{\text{put}} = \max (E - \frac{1}{n} \sum_{i=1}^{n} Price(t_i), 0) \] \tag{5}

Modifications of the basic Asian option model have been formulated, such as the American Asian option model [3]. This model allows the option to be exercised any time during its lifetime, keeping the payoff calculation using the mean price of the asset. However, this model is not widely extended, so we focus our work in the traditional Asian option model.

In year 1973 Fischer Black and Myron Scholes articulated a mathematical model capable of capturing the behaviour of a stock price movement [4]. This model, known as the Black Schole’s model, says that the price of a stock over time is governed by Eq.(6):

\[ S_{i+1} = S_i e^{(r - \frac{v^2}{2}) \Delta t + v \sqrt{\Delta t} W)} \] \tag{6}

where \( S_i = S(t_i) \) is the price of the asset at time step \( t_i \), \( r \) is the interest rate, \( v \) is the volatility of the underlying stock price, \( \Delta t \) is the time between two time steps and \( W \) is a Gaussian random number (GRN).

2.2. Monte-Carlo simulations

Monte-Carlo computations need to evaluate a large number of independent paths in order to converge to the solution. In our case this is translated into the need of a huge amount of Gaussian independent and identically distributed samples. When working with stock prices the quality of the Gaussian samples used becomes a critical matter: the very tiniest fraction of money is relevant. The more pseudo-random the Gaussian random number generator is, the more biased the simulation results will be, which could lead to relevant economic losses [5].

Most common methods for non-uniform RN generation are based on a first stage in which \( U[0,1] \) samples are generated [6]. Let \( u_i \) be a sample
of the $U[0,1]$ distribution, $X$ the target distribution and $ICDF$ its Inverse Cumulative Distribution Function. The inversion method states that

$$z_i = ICDF(u_i)$$

is a sample of $X$. Even direct methods which claim to directly generate non-uniform RN such as [7],[8] use a uniform RN generator in their core.

Along the rest of this section, the methodology used to generate high-quality GRN will be described, separated in two steps: generating uniform samples and using these uniform samples to generate Gaussian ones. At the end of the section our approach will be placed in contrast with the previously developed works on this matter.

2.2.1. Mersenne Twister

As it has already been stated, a high quality uniform pseudo-random generator is required. There are several key properties to measure the goodness of such generator [9, 10]:

- its period $p$. This is a determining parameter not only in order to avoid the repetition of pseudo-random sequences, but also in order to be able to run larger simulations. As explained in [10], different authors sustain that the number of values to be used in an experiment using a pseudo-random generator with period $p$ must be much lower than $p$ itself: some say that $p/100$ is enough, but others defend that even lower amounts of numbers must be used, such as $\sqrt{\frac{p}{200}}$ or even $\sqrt[3]{p}$.

- its $k$-dimensional equidistributed property\(^1\)

However, when working with hardware devices, there is another key factor that must be taken into account when using a RNG:

- the amount of area (in terms of memory and other silicon resources) required by the RNG. There are several RNG with have a very large

\(^{1}\)A pseudorandom sequence $x_i$ of $w$-bit integers of period $P$ is said to be $k$-distributed to $v$-bit ($v$ lew) accuracy if for each subsequence

$$(\text{trunc}_v(x_i), \text{trunc}_v(x_{i+1}), \ldots, \text{trunc}_v(x_{i+k-1})) \quad 0 \leq i \leq P$$

each of the $2^{kv}$ possible combinations of bits occurs the same number of times in a period, except for the all-zero combination that occurs once less often.
period due to the use of a bigger amount of memory keeping the RNG state information.

Table 1 shows some of the features for several RNG that have been evaluated during the development of this work. This table shows as well the results obtained when applying a suite of difficult-to-pass uniformity tests [11] to such generators. Those tests has been run with more than $5.6 \times 10^9$ values generated for each RNG. Mersenne Twister was chosen amongst the considered RNGs due to their good results and its period-memory tradeoff.

Table 1: Properties of different RNGs

<table>
<thead>
<tr>
<th>RNG</th>
<th>memory (32b words)</th>
<th>$\log_2(\text{Period})$</th>
<th>% of tests passed</th>
<th>Refs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>taus88</td>
<td>3</td>
<td>88</td>
<td>96.49</td>
<td>[12]</td>
</tr>
<tr>
<td>mcw256</td>
<td>256</td>
<td>8222</td>
<td>97.37</td>
<td>[10]</td>
</tr>
<tr>
<td>lr</td>
<td>345</td>
<td>$&gt; 345$</td>
<td>97.37</td>
<td>[13]</td>
</tr>
<tr>
<td>mt19937</td>
<td>624</td>
<td>19937</td>
<td>99.12</td>
<td>[9]</td>
</tr>
<tr>
<td>sobol (40-dim)</td>
<td>1280</td>
<td>32</td>
<td>98.25</td>
<td>[14, 15, 16]</td>
</tr>
<tr>
<td>cmwc4096</td>
<td>4096</td>
<td>131086</td>
<td>97.37</td>
<td>[10]</td>
</tr>
<tr>
<td>superKiss</td>
<td>41790</td>
<td>1337279</td>
<td>99.12</td>
<td>[10]</td>
</tr>
</tbody>
</table>

The Mersenne Twister (MT) is a high quality uniform random number generator with a period of $2^{19937} - 1$ and a 623-dimensional equidistributed property. The original C source code can be found in [9] and its CPU execution is highly efficient, both in terms of memory and performance. A matrix of 624 32-bit integers is initialized in a first stage. After that, one value of the matrix is returned with each call to \texttt{genrand()} through some bit manipulation process known as \textit{tempering}. Once all the values in the matrix have been used, it is reconstructed by shuffling and masking its bits.

Although the Mersenne-Twister code behaves properly on CPU computing, the lack of constant throughput (because of shuffling and masking the generation matrix) of this approach turns it into a quite pipeline-unfriendly code. It is important to remark that we have decided to use such a pipeline-unfriendly approach for generating random uniform samples in contrast with other HW-friendlier existing approaches [17]. We have made this decision because accuracy has been considered a much more critical issue rather than
throughput for our Asian option simulation solution. We consider that our approach presents a more appropriate accuracy-throughput tradeoff than other existing solutions such as [18], [19] or [20]. Moreover, previous works such as [21] shows that despite its pipeline-unfriendliness, MT can be highly benefited using FPGA technology.

2.2.2. Box-Muller transform

For our Asian option pricing problem we are interested in generating Gaussian samples, whose $CDF$ is given by Eq.(7).

$$CDF(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}} dt \tag{7}$$

As there is no closed form for its $ICDF$ function, the generic inverse method can not be applied. Some approaches have been developed in previous works such as [19] or [20] in which a approximation to the $ICDF$ function is used. A tradeoff must be met between the accuracy requirements and the complex expressions that need to be evaluated in order to meet these accuracy constraints. These expressions require a substantial amount of floating-point operations so their implementation would severely restrict the number of Monte-Carlo cores that could be mapped inside a FPGA.

Fortunately, there is another method for generating Gaussian samples from uniform samples, which is the Box-Muller transformation [22]. It requires less floating-point arithmetic, and its good quality has already been proved [23].

The Box-Muller transformation produces two independent Gaussian samples from two independent uniform ones. Let $u_1$ and $u_2$ be two independent samples of the $U[0, 1]$ distribution, then

$$z_0 = \sqrt{-2 \ln u_1 \cos(2\pi u_2)}$$
$$z_1 = \sqrt{-2 \ln u_1 \sin(2\pi u_2)}$$

are two independent Gaussian samples $\sim \mathcal{N}(0, 1)$.

The implementation of this transform with Impulse C is trivial in a pipelined process, generating one sample per clock cycle. The only issue is the area required by the transcendental and trigonometric floating-point operators from the Altera Megafunctions [24], which limits the number of Monte-Carlo cores that can be implemented in the FPGA. In order to slightly
reduce this area utilization, the identity $\sin^2\alpha + \cos^2\alpha = 1$ is utilized, saving one trigonometric evaluation at the cost of adding one square root and one multiplication.

2.3. Related work

As for other financial problems, there has been a lot of effort made in the resolution of Asian option pricing. Other approaches rather than Monte-Carlo methods have been researched [25], but the parallelizability of Monte-Carlo simulation has made this approach to be much widely extended in the High-Performance Computing world.

However, the use of Monte-Carlo simulation methods for this calculation makes this task a heavy computational one, so many high performance computing approaches has been developed to solve this problem. Some of those solutions are based on message passing programming [26] [27] for high performance clusters. Nevertheless, the most notable efforts have been made in the field of reconfigurable/reprogrammable coprocessor computing, both based on FPGA [18] [19] [28] and GP-GPU [19] [29] technologies.

All those works have focused their efforts in creating a high-performance simulation solution, giving priority to the throughput of the system rather than to its accuracy. In this work, we have tried to create an as accurate as possible solution for the Asian option pricing problem, keeping in mind the relevance of the timing behaviour of the system.

Table 2 summarizes the performance results shown in the mentioned related works, as well as the technology approach used for each solution and their performance reference.

3. High Level Languages

The use of High Level Languages has a set of advantages against the use of low level programming languages. The most indisputable of those advantages is the decrease of the development time. Thanks to that, a fast evaluation of the feasibility of a project via prototyping becomes a reality. As previous works has shown [30], [31], [32] this effect becomes more evident when it comes to programming FPGA devices in contrast with hardware description languages (VHDL, Verilog).

Of course, HLL-based FPGA design leads to less area-efficient design in contrast with HDL-based design. A tradeoff must be met regarding the development time and the area efficiency constraints for each design.
<table>
<thead>
<tr>
<th>Ref.</th>
<th>Year</th>
<th>Technology</th>
<th>Uniform</th>
<th>RNG</th>
<th>Gaussian</th>
<th>Speedup</th>
</tr>
</thead>
<tbody>
<tr>
<td>[19]</td>
<td>2011</td>
<td>Xilinx Virtex-5</td>
<td>lr</td>
<td>ICDF piecewise interpolation</td>
<td>313.00 vs Intel Xeon 2.5 GHz</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Nvidia Tesla C1060</td>
<td>mt19937</td>
<td>Box-Muller</td>
<td>141.00 vs Intel Xeon 2.5 GHz</td>
<td></td>
</tr>
<tr>
<td>[26]</td>
<td>2011</td>
<td>2x Intel dual-core 1.6 GHz</td>
<td>lr</td>
<td>Box-Muller</td>
<td>2.13 vs Intel dual-core 1.6 GHz</td>
<td></td>
</tr>
<tr>
<td>[29]</td>
<td>2010</td>
<td>Nvidia Quadro FX4600</td>
<td>sobol</td>
<td>ICDF approximation</td>
<td>150.00 vs Quad-Core Xeon</td>
<td></td>
</tr>
<tr>
<td>[28]</td>
<td>2008</td>
<td>Xilinx Virtex-4</td>
<td>taus88</td>
<td>Box-Muller</td>
<td>340.00 vs Intel Xeon 2.8 GHz</td>
<td></td>
</tr>
</tbody>
</table>
3.1. Impulse C

The development of this work has been made using the Impulse C programming framework [33]. The choice of this language has been made because of its outstanding usability compared to other high-level approaches for FPGA programming, as stated in [30]. Moreover Impulse C potential has been shown in previous works such as [34], [35] and [36].

Impulse C extends standard ANSI-C using C-compatible predefined library functions in support of a communicating process parallel programming model. This programming model is conceptually similar to a dataflow or a communicating sequential process programming model in the sense that it simplifies the expression of highly parallel algorithms through the use of well-defined data communication, message passing, and synchronization mechanisms. The programming model supports a wide range of applications and parallel process topologies.

In Impulse C, the programming model emphasizes the use of buffered data streams as the primary method of communication between independently synchronized processes, which are implemented as persistent (rather than being repetitively called) C subroutines. This buffering of data, which is implemented using FIFOs that are specified and configured by the application programmer, makes it possible to write parallel applications at a higher level of abstraction, without the clock cycle-by-cycle synchronization that would otherwise be required.

Programming with Impulse C processes is conceptually similar to programming with threads. As with thread programming, each Impulse C process has its own control flow, it is independently synchronized and it has access to its own local memory resources (which will vary depending on the target platform). For this reason it is relatively easy to convert applications written in threaded C (for example, using the Posix thread library [37]) to Impulse C. Thus, the coarse parallelism in Impulse C is simply achieved by coding multiple concurrent processes. Synchronization among these concurrent computational kernels is easily achieved by the stream dataflow [38].

Finer levels of parallelism can be exploited inside each process. Instruction level parallelism is automatically generated by the scheduler. Impulse C automatically generates and analyses the instruction dependence graph so that independent instructions are scheduled in parallel. In the case of loops, further level of parallelism can be achieved by means of precompiler directives or pragmas. They are loop unrolling and loop pipelining, which
are specified by placing \#pragma CO UNROLL and \#pragma CO PIPELINE just after the header of a loop.

An additional feature of the Impulse C framework is the ability to easily modify the IP cores to be instantiated whenever a hard-function or an arithmetic operator appears in the C source code. This is made by editing a set of XML files attached to the target platform. The Impulse C vendor refers to this feature as Platform Support Package (PSP). The ease-of-use of the PSP turns the Impulse C framework into an extremely interesting option when working with custom cores, so anyone can create their own optimized HDL-based core and use C code to instantiate it along the design. The use of PSPs strengthens as well the portability of a project between different platforms.

4. Implementation

4.1. Option price calculation

The Monte-Carlo algorithm for pricing Asian Options is depicted in Listing 1. For each simulation path, stock prices are updated by using Equation (6) and accumulated in order to compute the mean value until the maturity time $T$. The payoff is evaluated as the profit involved in the transaction. Again, values are accumulated in order to obtain the mean value. Finally, the price is adjusted by the fixed interest rate. The architecture of the solution is shown in Fig. 1 where each of the functional modules are identified, as well as the FIFO channels that Impulse C instantiates for inter-module communication.

In order to accelerate this algorithm with Impulse C, loop pipelining must be used. This is trivially achieved by simply inserting \#pragma CO PIPELINE after the head of the body. However, Impulse C does not allow pipeline nesting and the two loops should be flattened into a single loop executing $nSim \times nSteps$ iterations. However, there is loop dependence between iterations which prevents a pipeline with rate 1, i.e, a pipeline computing iterations every clock cycle. To avoid the loop dependence, we must take advantage of the independence of the stock prices among different simulations, so that PARALLEL_PATHS simulations are concurrently computed.

To this end we split the problem in two different Impulse C processes. The first one computes the stock prices and performs the arithmetic mean. It also computes the payoff, which is written to the second process. Listing 2 depicts the Impulse C pseudo-code working at rate 1. There are two different
arrays of PARALLEL_PATHS elements which store the stock and the addition until the $i$-th step, respectively. We use the CO NONRECURSIVE pragma to tell Impulse C that there are no data dependences regarding the array access, so that waits are not inserted. However, we must set PARALLEL_PATHS big enough so that the previous assertion is actually true. The optimum value for PARALLEL_PATHS would be the latency of the critical code in between the read from and the write to the memories. As PARALLEL_PATHS is going to be small and it does not affect performance, it is easier to simply use the total routine latency (85) as the lower bound, since it is automatically reported by Impulse C. Therefore, with the rate 1 pipeline, all the payoffs are computed after $nSteps \times nSims$ plus the pipeline latency, which is despicable.

Each of these computed payoffs must be added, that is the function of the second Impulse C processes. A naive implementation would be the one in Listing 3. Here again there is the problem of loop dependence and the pipeline can only perform at a rate equal to the latency of the addition operator. A simple trick can be used to avoid this inconvenient. The accumulation is performed in different positions of an array with a size BUFF_SIZE greater or equal to the addition latency, as shown in Listing 4. This way, the additions can be performed with rate 1 plus a negligible penalization to add the final values stored in the array. As before, we must only ensure that BUFF_SIZE is bigger than the pipeline latency so that it is true that there is no dependence between all the stages in the pipeline.

Therefore the extra time needed for adding the $nSims$ values would be
Listing 1: Monte-Carlo algorithm for pricing Asian Options

```c
payOffSum = 0;
for (i = 0; i < nSims; i++){
    priceSum = $S_0$;
    for (j = 0; j < nSteps; j++){  
        W = getGaussian();
        S = $S \times \exp((r - v^2/2) \Delta t + v \times W \times \sqrt{\Delta T})$;
        priceSum += S;
    }
    payOff = max(0, priceSum /(nSteps+1) - K);
    payOffSum += payOff;
}
price = \exp(-r \times T) \times \text{payOffSum}/nSims;
```

approximately of $nSims + l_a + \text{BUFF\_SIZE} \cdot l_a \approx nSims$ clock cycles, where $l_a$ is the addition latency. However, the overall computation time of our Monte-Carlo core is essentially of $nSteps \times nSims$ cycles, since there is overlapping between the payoff computation and the sum reduction.

### 4.2. Random number generation

The traditional Mersenne-Twister CPU code is not HW-friendly because it lacks a constant throughput due to the shuffling and masking stages made once all of the values in the matrix have been used. To overcome this limitation we have rewritten the algorithm so that once the value of the matrix is used, it is replaced by the new one. Moreover, the MT algorithm reads two data and writes one for each random number generation. Since the matrix is mapped into a BlockRAM, whose simultaneous access is limited to a maximum of two ports, we need two different matrices to achieve a one sample per clock cycle throughput. After initialization, the matrix A takes the read-only role. Its values are tempered and returned as the uniform samples, one each clock cycle. At the same time, the matrix B is filled with a new Mersenne Matrix. Once all the values in the matrix A have been used, the roles are interchanged so that B becomes read-only.

A Mersenne Twister core with a throughput of 623 samples per clock cycle has also been developed by implementing the Mersenne matrix with registers. This is somehow a tedious task when working with Impulse C,
Listing 2: Impulse C pseudo-code for the payoff computation in a pipeline with rate 1

```c
for (i = 0; i < nSteps*nSims; i++){
    #pragma CO PIPELINE
    #pragma CO NONRECURSIVE lastStockPrice
    #pragma CO NONRECURSIVE lastStockSum

    co_stream_read(sGauss, &W, sizeof(float));

    if (initStage(i)){
        S = S0;
        priceSum = S0;
    } else{ // Accumulation stage
        S = lastStockPrice[i % PARALLEL_PATHS];
        priceSum = lastStockSum[i % PARALLEL_PATHS];
    }
    S = updateStock(W, S);
    priceSum += S;
    lastStockSum[i % PARALLEL_PATHS] = priceSum;
    lastStockPrice[i % PARALLEL_PATHS] = S;

    if (outputStage(i)){
        payOff = computePayOff(priceSum, K);
        co_stream_write(sPayOff,&payOff, sizeof(float));
    }
}
```
Listing 3: Impulse C pseudo-code for the payoff accumulation in a pipeline with a rate equal to the latency of the addition operation (7 cycles for the lowest latency Altera Megafunction)

```
1 payOffSum = 0;
2 for ( i = 0; i < nSims; i++){
3     #pragma CO PIPELINE
4     co_stream_read(sPayoffIn, &payOff, sizeof(float));
5     payOffSum += payOff;
6 }
```

as there is no way to tell the tool that we want a matrix inferred this way. Instead, we declared each element of the matrix as a different 32-bit unsigned integer variable and we generated the Impulse C code with the help of a python script. However, the area required for this implementation was not worth the effort and it is preferable to instantiate several BlockRAM based Mersenne cores if more throughput is required.

Once one have been able to generate independent uniform random num-
ber using our Mersenne-Twister implementation, applying the Box-Muller transform to obtain Gaussian samples is a completely straightforward proce-
dure using Impulse C.

Note that in order to get unbiased simulation results, some care must be
taken with regard to the choice of the random seeds that will feed the differ-
ent Monte-Carlo simulation cores so the pseudo-random number sequences
generated by each simulation path are independent. Consequently, the seed-
ing must be made taking into account the principles detailed in [39, 40].
However, this seeding process must only be done once for each simulation,
so it can be done in software and the different seeds loaded into the FPGA
before starting the execution.

4.3. Floating-point arithmetic cores

Hardware acceleration requires high throughput, and pipelining is a must.
However, most pipelined intellectual property (IP) cores focus on working at
high clock frequencies, what in the end increases register utilization too much,
Listing 4: Impulse C pseudo-code for the payoff accumulation in a pipeline with rate 1

```
payOffSum = 0;
for (i = 0; i < nSims; i++){
    #pragma CO PIPELINE
    #pragma CO NONRECURSIVE sumsBuffer

    co_stream_read(sPayoffIn, &payOff, sizeof(float));

    if (initStage(i))
        payOffSum = 0;
    else{
        payOffSum = sumsBuffer[i % BUFF_SIZE];
    }

    payOffSum += payOff;
    sumsBuffer[i % BUFF_SIZE] = payOffSum;
}

// Final addition
payOffSum = 0;
for (i = 0; i < BUFF_SIZE; i++){
    payOffSum += sumsBuffer[i];
}
```
not only because of the internal registers of each core, but also because the registers needed to synchronize the signals in the pipeline.

Additionally, the high clock frequencies that these individual cores achieve are rarely reached in a whole design, due to I/O restrictions or routing restrictions. Therefore, the use of low-latency operators either reduces the utilization of FPGA resources by allowing the implementation of more complex algorithms or improves performance by allowing further parallelism by kernel replication.

Our arithmetic addition/subtraction and multiplication cores are fully compliant with the IEEE 754-2008 32-bit (single-precision) standard except for signaling NaNs and subnormal numbers. The former are only useful when collaborating with a microprocessor that handles exceptions, which is not typically the case in reconfigurable solutions. The latter are rarely implemented in FPGAs since the area and complexity involved in their implementation are not justified. By simply adding one more bit to the exponent, the numeric range outperforms by far that achieved by subnormals. Our single-precision arithmetic cores have been developed with the purpose of working at a moderate frequency of $\sim 100\text{MHz}$ at the lowest latency possible. Their area/timing characteristics are depicted in Table 3 in contrast with the standard Altera floating point cores [24].

<table>
<thead>
<tr>
<th>Operation</th>
<th>Core</th>
<th>Latency</th>
<th>ALUTs</th>
<th>Registers</th>
<th>DSPs</th>
<th>Mem bits</th>
<th>$F_{max}$ (Mhz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Add/Sub</td>
<td>Altera</td>
<td>7</td>
<td>581</td>
<td>373</td>
<td>0</td>
<td>0</td>
<td>284.5</td>
</tr>
<tr>
<td></td>
<td>Own</td>
<td>2</td>
<td>526</td>
<td>105</td>
<td>0</td>
<td>0</td>
<td>167.3</td>
</tr>
<tr>
<td>Mul</td>
<td>Altera</td>
<td>5</td>
<td>150</td>
<td>190</td>
<td>1</td>
<td>0</td>
<td>308.2</td>
</tr>
<tr>
<td></td>
<td>Own</td>
<td>1</td>
<td>154</td>
<td>147</td>
<td>1</td>
<td>0</td>
<td>287.8</td>
</tr>
<tr>
<td>Div</td>
<td>Altera</td>
<td>6</td>
<td>189</td>
<td>245</td>
<td>5</td>
<td>4608</td>
<td>195.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>14</td>
<td>200</td>
<td>702</td>
<td>5</td>
<td>4742</td>
<td>285.7</td>
</tr>
<tr>
<td></td>
<td>Own</td>
<td>7</td>
<td>1377</td>
<td>540</td>
<td>0</td>
<td>0</td>
<td>136.2</td>
</tr>
<tr>
<td>Sqrt</td>
<td>Altera</td>
<td>16</td>
<td>436</td>
<td>538</td>
<td>0</td>
<td>121</td>
<td>233.0</td>
</tr>
<tr>
<td></td>
<td>Own</td>
<td>7</td>
<td>435</td>
<td>298</td>
<td>0</td>
<td>0</td>
<td>132.4</td>
</tr>
<tr>
<td>Available</td>
<td></td>
<td></td>
<td>532720</td>
<td>1065440</td>
<td>1755</td>
<td>35942400</td>
<td></td>
</tr>
</tbody>
</table>

The development of those floating-point cores has been completely made using HDL-based design. However, the use of the Impulse C framework
allows to instantiate our cores whenever a floating-point arithmetic operation appears in the C source code. To that purpose, only some modifications must be made in the PSP files (which follow a XML syntax) corresponding to the target platform.

5. Results

The area utilization of the implementation discussed for a single computational kernel is shown in Table 4. These results are obtained with Quartus II 12.2 after Place and Route. The floating-point operators have been generated with the Megawizard Plug-in Manager from Altera. These operators are seamlessly integrated within Impulse C by editing some XML files. As it can be seen, the area bottleneck is found in the Box-Muller component, because of the area used by the trigonometric and logarithmic operators [24]. In the Stratix V 5SGSMD8, a total number of 40 computational Monte-Carlo kernels are instantiated. The outputs of these individual components are averaged by a collector Impulse C process which then performs the interest rate update in the price of the underlying asset.

<table>
<thead>
<tr>
<th>Core</th>
<th>ALUTs</th>
<th>Registers</th>
<th>DSPs</th>
<th>Mem bits</th>
<th>$F_{max}$ (Mhz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mersenne</td>
<td>744</td>
<td>867</td>
<td>2</td>
<td>40666</td>
<td>269.69</td>
</tr>
<tr>
<td>Box-Muller</td>
<td>8803</td>
<td>8155</td>
<td>22</td>
<td>1698</td>
<td>172.63</td>
</tr>
<tr>
<td>PayOffs</td>
<td>1242</td>
<td>1375</td>
<td>11</td>
<td>2268</td>
<td>206.48</td>
</tr>
<tr>
<td>SumPayOffs</td>
<td>1242</td>
<td>510</td>
<td>0</td>
<td>2498</td>
<td>156.48</td>
</tr>
<tr>
<td>1 Monte-Carlo core</td>
<td>9855</td>
<td>9409</td>
<td>29</td>
<td>41227</td>
<td>154.9</td>
</tr>
<tr>
<td>40 Monte-Carlo cores</td>
<td>396729</td>
<td>378509</td>
<td>1166</td>
<td>1682900</td>
<td>142.3</td>
</tr>
</tbody>
</table>

Performance results are depicted in Table 5 compared to several CPU implementations (for fairness reasons, the CPU version uses single-point floating operations). The CPU code is executed on Ubuntu 10.10 64 bit and the source is compiled with `gcc 4.4.5`, which allows the use of the OpenMP multicore programming framework [41], using the optimization flags `-O3` and
-ffast-math. By using the OpenMP framework, we allow the program to run through several computational cores just by placing the OMP PARALLEL FOR pragma before the outer loop of the Monte-Carlo simulation code. The CPU hardware consists of an Intel i7 860 CPU which runs at 2.8 GHz and it has 8 MB of cache.

For all the different versions the number of time steps, \( n_{\text{Steps}} \), is set to 3650 and the number of independent path simulations, \( n_{\text{Sims}} \), is \( 10^7 \). The FPGA execution time has been obtained by simulation but the performance in a real system will not suffer from communications, as data transfer time will be negligible compared to computation time. The tool reports a maximum frequency of 142.3 MHz, which leads to a total computation time of 6.41s. Comparing to the multi-core implementation, the reconfigurable implementation runs \( \sim 149 \) times faster (and \( \sim 504 \) faster than the single-core version).

<table>
<thead>
<tr>
<th></th>
<th>Time (s)</th>
<th>SpeedUp</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPU (1 core)</td>
<td>3232.60</td>
<td>1.00</td>
</tr>
<tr>
<td>CPU (2 cores)</td>
<td>1735.83</td>
<td>1.86</td>
</tr>
<tr>
<td>CPU (4 cores)</td>
<td>955.63</td>
<td>3.38</td>
</tr>
<tr>
<td>FPGA (40 cores)</td>
<td>6.41</td>
<td>504.07</td>
</tr>
</tbody>
</table>

The speedup obtained by our solution is comparable to the ones shown in the related work (see Table 2). A remark must be made again emphasizing the fact than keeping a comparable performance, our solution generates a better-quality amount of gaussian random number, which improves the Monte-Carlo simulation quality.

6. Conclusions

We have implemented a Monte-Carlo method for pricing Asian Options in FPGAs using floating-point arithmetic and Impulse C. The focus of this implementation has been made in the quality of the random number generation, a feature that has been obscured in the previous works in the best interest of performance. In contrast to that, our solution achieves a good accuracy-performance tradeoff.
With some tricks in the Impulse C code, and by exploiting path independence, we achieved pipelines with rate one both in the pseudo-random generator (Mersenne + BoxMuller) and in the Asian option pricing computation. The use of Impulse C language has allowed to obtain a good performance improvement in a drastically shorter period of time than a HDL-based development would have taken. Moreover, the use of the Impulse C framework has allowed to use some custom arithmetic in order to reduce the latency of the global system, as well as the total area of the simulation module.

As a result 40 Monte-Carlo cores have been located inside our Stratix V 5SGSMD8 Altera FPGA, showing a speed-up improvement of $\sim 504$ times over a single-core CPU implementation, and $\sim 149$ times over a 4-core one.

References


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