Photon Netw Commun
DOI 10.1007/s11107-008-0118-1

# Blocking models of optical burst switches with shared wavelength converters: exact formulations and analytical approximations

**Pedro Reviriego · Anna Maria Guidotti · Carla Raffaelli · Javier Aracil**

Received: 1 October 2007 / Accepted: 27 February 2008


**Abstract** Loss modeling of asynchronous optical burst switches with shared wavelength converters is considered. An exact analysis based on continuous time Markov chains is proposed and validated by comparison with simulation for balanced and unbalanced traffic. A computationally efficient approximated analysis is also proposed and compared with the exact model to find applicability conditions. Approximate loss performance evaluation is presented for ranges of values which are not tractable either by simulation or exact analysis.

**Keywords** Switching · OBS · Blocking · WDM

## 1 Introduction

Optical networks are a challenging solution to support the extremely high bandwidth requirements of next-generation telecommunication infrastructures. Particular interest in optical networking research is dedicated to flexible transfer modes to support bandwidth exploitation at different degrees of granularities and dynamicity. Packet-oriented techniques are the best candidates to achieve this goal although they call for optical switches which are able to effectively solve contention and reconfigure their interconnections very fast [1]. Optical Burst Switching (OBS) is a packet-based technique which has been widely studied in recent years and which could meet high speed networking requirements with adequate flexibility and fine granularity [2–4]. The implementation of Optical Burst Switched networks involves efforts in different areas, ranging from components to systems and related traffic models [1,5–7].

Different switch architectures can be considered to support optical burst switching. Typically these architectures provide wavelength conversion to solve the contention intrinsically related to this technique. The key components to perform this function are tuneable wavelength converters (TWCs) which are still complex components and in any case they represent a very expensive part of the optical burst switch [8]. As a consequence, attention is given to the design of switch architectures which share TWCs to perform wavelength conversion functions. Examples of these architectures are the share-per-link and the share-per-node architectures [9,10]. The system design of such architectures calls for efficient tools to evaluate loss performance and achieve optimized switch dimensioning. Some of these tools were proposed in literature [9]. Anyway numerically efficient algorithms are still needed to perform switch design in asynchronous contexts, with variable burst lengths, which better represent the optical burst switching behavior. This work aims at defining loss models for share-per-link and share-per-node architectures and compare them with approximated although numerically efficient models. In this paper, the switch is assumed to work asynchronously with variable burst length and with exponential distribution.

The paper is organized as follows. In Sect. 2 some reference to background and related works are given; in Sect. 3

P. Reviriego (✉)
Universidad Antonio de Nebrija, Madrid, Spain
e-mail: previrie@nebrija.es

A. M. Guidotti · C. Raffaelli
D.E.I.S.-University of Bologna, Bologna, Italy

A. M. Guidotti
e-mail: annamaria.guidotti@unibo.it

C. Raffaelli
e-mail: carla.raffaelli@unibo.it

J. Aracil
Universidad Autónoma de Madrid, Campus de Cantoblanco, Madrid, Spain
e-mail: javier.aracil@uam.es

the architectures considered are described. In Sect. 4 the proposed exact analytical models for loss evaluation are described. In Sect. 5 an approximated model is introduced. In Sect. 6 the validation of the exact model by simulation and the comparisons between exact and approximated models are presented and discussed. In Sect. 7 conclusions of the work are drawn.

## 2 Background and related works

Optical Burst switching is a networking technique designed to be employed in WDM optical networks. Wavelength division multiplexing (WDM) is a transmission scheme where many independent wavelength channels are transmitted across a large network. It utilizes the available fiber bandwidth at wavelength granularity, by increasing the aggregate system capacity and throughput of raw fiber networks [11]. To further exploit the bandwidth in the presence of dynamically changing requirements each channel can be further shared by bursts, which are formed by collection of packets from peripheral networks that are aggregated at the optical network edge according to burst assembly algorithms [12]. Burst traffic generated at network edges is transferred through the optical burst switched network through suitable scheduling of interconnection links in optical burst switches [13]. The characteristics of this traffic have been studied in some previous works [14–17]. Some works also propose switch architectures suitable to support this kind of traffic [9,18]. In any case, we focus here on forwarding operation of bursts in optical switches. In this view the study can be applied both at optical burst or packet switched networks, providing that asynchronous context is assumed. For this reason the term packet or burst can be interchangeably used.

The design of the WDM switches with simple and cost effective components is a key issue in optical networking. A general scheme of an optical node is composed by a demultiplexing stage, where all wavelengths in the fiber are split and information is drawn from each wavelength and considered individually by the control unit. A non-blocking optical stage is needed afterward to forward packets to the right destination. At the output stage wavelengths are again multiplexed on the same physical support and eventually information is sent. To improve the performance, especially in a packet-oriented environments, TWCs and optical buffers can be added to the aforementioned structure. Wavelength conversion in optical networks is considered here to perform contention resolution by application of tunable wavelength converters [19]. Bursts are shifted from a wavelength to another to achieve efficient bandwidth sharing. Anyway wavelength conversion is not easy from the technology point of view. Experimental results have shown that performance of wavelength converters strongly depends on combination of the input and output wavelengths. That is, for a given input wavelength, translations to some output wavelengths result in an output signal which is significantly degraded [10]. Moreover, the wider the range that a converter has to work with the more expensive it results.

Multiple fibers can be used to alleviate wavelength conversion. The multi-fiber solution seems to suit with this aspect. This scheme was already explored for wavelength switching networks [20]. The investigation of this approach for optical packet switching in asynchronous networks is rather new. A reason to take into account this structure is that a large number of fibers are already contained in a cable underground so no further digging would be necessary. Furthermore, multi-fiber proves to be efficient either in terms of performance and conversion cost.

Loss models for optical packet switches with shared wavelength converters have been proposed in the literature with reference to different switch architectures. In this paper we focus on asynchronous switches with wavelength converters share-per-link and share-per-node, possibly with multi-fiber configurations. Exact models for asynchronous context typically assume exponential burst length distribution and are based on Markov chain approaches [21]. These kind of models, although in some cases lead to exact solutions, typically put large demand on memory space as the switch size increases, making the approach of limited applicability in practice. Thus, there is a need for approximated and computationally feasible approaches.

First we present an exact loss probability analysis for the architecture under study, which is validated by simulation. An approximated approach is then presented and compared with the exact one to understand its effectiveness.

## 3 Node architectures

Two architectures are proposed that implement different schemes for wavelength converters' sharing. The first one applies the share-per-link policy and is sketched in Fig. 1. It employs as many pools of converters as the number of output interfaces, each shared among the wavelength channels belonging to the same interface. The architecture presented in Fig. 2 applies the share-per-node option. A single pool of converters is available and shared among all node channels.

The external setting is the same for both architectures. It consists of $N$ inputs and $N$ outputs, equipped with $F$ fibers carrying $M$ wavelengths each. This configuration provides $F \times M$ wavelength channels per output interface. In the first case (share-per-link) $R$ indicates the number of TWCs that are available to wavelength channels switched to the corresponding interface. In the second case (share-per-node) $C$ represents the number of converters that belong to the single pool shared among all node's channels. In both cases

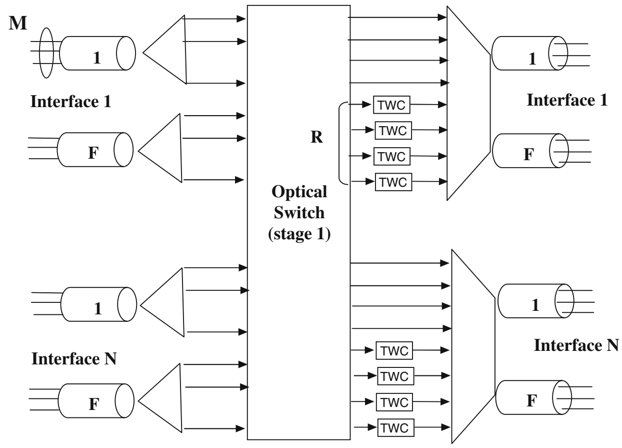


**Fig. 1** Switching node architecture with $N$ input/output ports, $M$ wavelengths per fiber, and a set of $R$ share-per-link wavelength converters

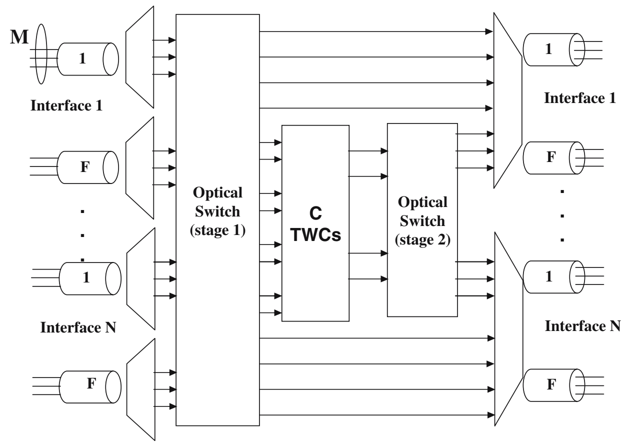


**Fig. 2** Switching node architecture with $N$ input/output ports, $M$ wavelengths per fiber, and a set of $C$ share-per-node wavelength converters

a set of links without converters is also provided to forward packets that do not need conversion. Looking at the architecture from left to the right, the general switch behavior can be described as follows: in the de-multiplexing phase channels are separated at the input ports and then kept separated until they will be again multiplexed at output ports. After the de-multiplexing phase the first optical switch selects the proper output interface which is identified by the switch control unit on the basis of the packet destination address. The packet might be sent to the converters' pool or not depending on the need of wavelength conversion. A second switch selects the right fiber within the interface. The first optical switch stage, as presented in Figs. 1 and 2, is quite large, being it ($N \cdot F \cdot M \times N \cdot F \cdot M$). To overcome this problem, this stage can be organized into parallel planes, one for each wavelength employed, thus reducing the required size of each plane to ($N \cdot F \times N \cdot F$) providing additional de-multiplexing and multiplexing functions [22]. A good compromise between efficiency and feasibility is fundamental when designing such architectures. As it will be shown later, the higher the number of fibers $F$ is, the better the switch performs. But increasing $F$ means also increasing the number of other components as Mux/Demux. For a matter of space and complexity these components cannot be too many within a single switch so a good trade-off must be reached. As regards the wavelengths' assignment to fiber at a given interface, the following different solutions can be adopted:

1. the $F \times M$ wavelengths used at the switch interface are all different;
2. the same set of $M$ wavelengths is repeated on each of the $F$ fibers.

In case 2, converters need to work within a narrower band compared with case 1 or even compared with the single fiber per interface option where all wavelengths are necessarily distinct. Consequent feasibility and cost reduction can be so achieved [22] and this solution is adopted here.

## 4 Exact analytical models

In this section, we provide an exact analytical model for the blocking probability calculation. Such model is based on a Continuous-Time Markov Chain (CTMC) and thus assumes Poisson arrivals (rate $\lambda$) and exponential service times (average service time $1/\mu$). It is only feasible for a small number of ports ($N$), fibers ($F$), wavelengths per fiber ($M$), and converters ($R$ or $C$), since the matrix dimension becomes too large. We consider the following cases

- *Share-per-link*
  - One fiber per port ($F = 1$), $M$ wavelengths and $R$ converters ($R \leq M$).
  - Two fibers per port ($F = 2$), $M$ wavelengths and $R$ converters ($R \leq 2 * M$).
- *Share-per-node*
  - One fiber per port ($F = 1$), $M$ wavelengths and $C$ converters ($C \leq N * M$) shared between two ports $N = 2$.

Additionally, it is worth remarking that throughout the analysis we will be assuming equally likely destinations, which allows us to focus on a single link when appropriate.

### 4.1 Share-per-link: One fiber per port ($F = 1$), $M$ wavelengths and $R$ converters ($R \leq M$)

Let us consider the bi-dimensional CTMC $\{(m(t), r(t)), t \geq 0\}$, which, for notation simplicity will be denoted by $(m, r)$, where $m$ represents the number of busy wavelengths (out of $M$) and $r$ represents the number of busy wavelengths converters (out of $R$). Note that $0 \le m \le M$, $0 \le r \le R$, and $r \le m$ so that the possible values for $(m, r)$ are given in the following matrix:

$$\mathbf{S} = \begin{pmatrix} (0,0) & * & \dots & & & * \\ (1,0) & (1,1) & * & \dots & & * \\ (2,0) & (2,1) & (2,2) & * & \dots & * \\ \dots & \dots & \dots & \dots & \dots & \dots \\ (R,0) & (R,1) & (R,2) & \dots & (R,R) & * \dots * \\ \dots & \dots & \dots & \dots & \dots & \dots \\ (M-1,0) & (M-1,1) & (M-1,2) & \dots & (M-1,R) & * \quad * \\ (M,0) & (M,1) & (M,2) & \dots & (M,R) & * \quad * \end{pmatrix} \quad (1)$$

From (1) we note that the states that contribute to the blocking probability are the ones in the bottom row and $R$th column. In the first case, $(M, r), r = 0, \dots R$, blocking is due to outage of outgoing wavelengths. In the second case, $(m, R), m = R, \dots M$, blocking is due to outage of wavelength converters. In the latter case the blocking probability is equal to $m/M$ that accounts for new requests to an already occupied lambda, with no wavelength converters available. Let us denote by $p(m, r)$ the steady-state probability for the $(m, r)$ chain. Then, the blocking probability $B$ is given by

$$B = \sum_{r=0}^{R} p(M, r) + \sum_{m=R}^{M-1} p(m, R) \frac{m}{M} \quad (2)$$

To obtain the blocking probability, we follow the usual methodology for CTMCs. We observe that $(m, r)$ is aperiodic and irreducible. Let $\mathbf{Q}$ be the infinitesimal generator for the chain. Let $\mathbf{P}$ denote the steady-state probability vector $(m, r)$. Then, we obtain $p(m, r)$ by applying

$$\mathbf{PQ} = \mathbf{0}$$
$$\sum_{(m,r)\in\mathbf{S}} p(m, r) = 1 \quad (3)$$

where $\mathbf{S}$ is given by (1). Let $p'_{(i,j)(i',j')} = \mathrm{d}p_{(i,j)(i',j')}/\mathrm{d}t$ denotes the transition *rate* from state $(i, j)$ to state $(i', j')$. Then $\mathbf{Q} = \left(p'_{(i,j)(i',j')}\right)$. Table 1 represents the transition rates between states in $\mathbf{S}$.

Note that the total number of states in (1) is equal to $(R+1)(R+2)/2 + (M-R) * (R+1)$, which is $\mathcal{O}(M)$, assuming $R \ll M$.

### 4.2 Share-per-link: Two fibers per port ($F = 2$), $M$ wavelengths and $R$ converters ($R \le 2 * M$)

Since we have two fibers per wavelength we note that the state of a given wavelength may be

- $s_1$: Wavelength is not in use in any of the fibers.
- $s_2$: Wavelength is busy in one of the fibers, no converter is used.
- $s_3$: Wavelength is busy in both the fibers, no converter is used.
- $s_4$: Wavelength is busy in one of the fibers, one converter is used.
- $s_5$: Wavelength is busy in both the fibers, one converter is used.
- $s_6$: Wavelength is busy in both the fibers, two converters are used.

To calculate the blocking probability, we only need to know the number of wavelengths that are in each of the above states. Thus, the CTMC is now defined by the 6-tuple $(m_{s_1}, m_{s_2}, m_{s_3}, m_{s_4}, m_{s_5}, m_{s_6})$ where $m_{s_i}$ denotes the number of wavelenghts in each possible state and $m_{s_1} + m_{s_2} + m_{s_3} + m_{s_4} + m_{s_5} + m_{s_6} = M$, $m_{s_4} + m_{s_5} + 2 * m_{s_6} \le R$.

The policy for using converters is as follows. If a packet arrives for a wavelength that is in any of the "Busy" states ($m_{s_3}$, $m_{s_5}$, and $m_{s_6}$) a converter is used with the following conventions:

1. If there are available wavelengths with state $s_1$ then one of them is selected.
2. If no wavelengths are available with state $s_1$ and there are wavelengths available with state $s_2$ then one of latter is randomly selected.
3. If no wavelengths are available neither in state $s_1$ nor in state $s_2$ but there are available wavelengths in state $s_4$ then one of the latter is randomly selected.
4. Blocking occurs if all wavelengths are in "Busy" states ($m_{s_3}$, $m_{s_5}$, and $m_{s_6}$).

Table 2 shows the transition rates from $(m_{s_1}, m_{s_2}, m_{s_3}, m_{s_4}, m_{s_5}, m_{s_6})$ to $(m'_{s_1}, m'_{s_2}, m'_{s_3}, m'_{s_4}, m'_{s_5}, m'_{s_6})$. For space limitations we omit the allowable ranges for $(m_{s_1}, m_{s_2}, m_{s_3}, m_{s_4}, m_{s_5}, m_{s_6})$.

The last three rows are valid if and only if there are converters available. Finally, the blocking probability is given by

$$B = \sum_{\substack{m_{s_4}+m_{s_5}+2*m_{s_6}=R; \\ m_{s_3}+m_{s_5}+m_{s_6}\neq M}} \frac{(m_{s_3} + m_{s_5} + m_{s_6})}{M} \times p(m_{s_1}, m_{s_2}, m_{s_3}, m_{s_4}, m_{s_5}, m_{s_6}) + \sum_{m_{s_3}+m_{s_5}+m_{s_6}=M} p(m_{s_1}, m_{s_2}, m_{s_3}, m_{s_4}, m_{s_5}, m_{s_6}) \quad (4)$$

where $p(\cdot)$ are steady-state probabilities. The first summation accounts for the blocking due to all converters being busy, while the second one accounts for the blocking due to all wavelengths being busy on both fibers.

**Table 1** Transition rates between the states in **S**

| $i$ | $j$ | $i'$ | $j'$ | Rate |
|---|---|---|---|---|
| $2 \le i \le M-1$ | $1 \le j \le R-1,\ j \le i$ | $i+1$ | $j$ | $(1-i/M)\lambda$ |
| $2 \le i \le M$ | $1 \le j \le R-1,\ j \le i$ | $i-1$ | $j$ | $(i-j)\mu$ |
| $2 \le i \le M-1$ | $1 \le j \le R-1,\ j \le i$ | $i+1$ | $j+1$ | $(i/M)\lambda$ |
| $1 \le i \le M$ | $1 \le j \le R-1,\ j \le i$ | $i-1$ | $j-1$ | $j\mu$ |
| $1 \le i \le M$ | $0$ | $i-1$ | $0$ | $i\mu$ |
| $M$ | $0 \le j \le R$ | $M-1$ | $j$ | $(i-j)\mu$ |
| $M$ | $0 \le j \le R$ | $M-1$ | $j-1$ | $j\mu$ |
| $R \le i \le M$ | $R$ | $i-1$ | $j-1$ | $j\mu$ |
| $R \le i \le M-1$ | $R$ | $i+1$ | $j$ | $(1-i/M)\lambda$ |
| $R+1 \le i \le M$ | $R$ | $i-1$ | $j$ | $(i-j)\mu$ |
| $1 \le i \le R$ | $1 \le j \le R,\ j=i$ | $i+1$ | $j$ | $(1-(i/M))\lambda$ |
| $1 \le i \le R$ | $1 \le j \le R,\ j=i$ | $i-1$ | $j-1$ | $i\mu$ |
| $0$ | $0$ | $1$ | $0$ | $\lambda$ |

**Table 2** Transition rates between the states in the share-per-link case with $F = 2$

| $m_{s_1}$ | $m_{s_2}$ | $m_{s_3}$ | $m_{s_4}$ | $m_{s_5}$ | $m_{s_6}$ | $m'_{s_1}$ | $m'_{s_2}$ | $m'_{s_3}$ | $m'_{s_4}$ | $m'_{s_5}$ | $m'_{s_6}$ | Rate |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $m_{s_1}$ | $m_{s_2}$ | $m_{s_3}$ | $m_{s_4}-1$ | $m_{s_5}$ | $m_{s_6}+1$ | $m_{s_1}$ | $m_{s_2}$ | $m_{s_3}$ | $m_{s_4}$ | $m_{s_5}$ | $m_{s_6}$ | $2\mu(m_{s_6}+1)$ |
| $m_{s_1}$ | $m_{s_2}$ | $m_{s_3}$ | $m_{s_4}-1$ | $m_{s_5}+1$ | $m_{s_6}$ | $m_{s_1}$ | $m_{s_2}$ | $m_{s_3}$ | $m_{s_4}$ | $m_{s_5}$ | $m_{s_6}$ | $\mu(m_{s_5}+1)$ |
| $m_{s_1}$ | $m_{s_2}-1$ | $m_{s_3}$ | $m_{s_4}$ | $m_{s_5}+1$ | $m_{s_6}$ | $m_{s_1}$ | $m_{s_2}$ | $m_{s_3}$ | $m_{s_4}$ | $m_{s_5}$ | $m_{s_6}$ | $\mu(m_{s_5}+1)$ |
| $m_{s_1}-1$ | $m_{s_2}$ | $m_{s_3}$ | $m_{s_4}+1$ | $m_{s_5}$ | $m_{s_6}$ | $m_{s_1}$ | $m_{s_2}$ | $m_{s_3}$ | $m_{s_4}$ | $m_{s_5}$ | $m_{s_6}$ | $\mu(m_{s_4}+1)$ |
| $m_{s_1}$ | $m_{s_2}-1$ | $m_{s_3}+1$ | $m_{s_4}$ | $m_{s_5}$ | $m_{s_6}$ | $m_{s_1}$ | $m_{s_2}$ | $m_{s_3}$ | $m_{s_4}$ | $m_{s_5}$ | $m_{s_6}$ | $2\mu(m_{s_3}+1)$ |
| $m_{s_1}-1$ | $m_{s_2}+1$ | $m_{s_3}$ | $m_{s_4}$ | $m_{s_5}$ | $m_{s_6}$ | $m_{s_1}$ | $m_{s_2}$ | $m_{s_3}$ | $m_{s_4}$ | $m_{s_5}$ | $m_{s_6}$ | $\mu(m_{s_2}+1)$ |
| $m_{s_1}+1$ | $m_{s_2}-1$ | $m_{s_3}$ | $m_{s_4}$ | $m_{s_5}$ | $m_{s_6}$ | $m_{s_1}$ | $m_{s_2}$ | $m_{s_3}$ | $m_{s_4}$ | $m_{s_5}$ | $m_{s_6}$ | $\lambda(m_{s_1}+1)$ |
| $m_{s_1}$ | $m_{s_2}+1$ | $m_{s_3}-1$ | $m_{s_4}$ | $m_{s_5}$ | $m_{s_6}$ | $m_{s_1}$ | $m_{s_2}$ | $m_{s_3}$ | $m_{s_4}$ | $m_{s_5}$ | $m_{s_6}$ | $\lambda(m_{s_2}+1)$ |
| $m_{s_1}$ | $m_{s_2}$ | $m_{s_3}$ | $m_{s_4}+1$ | $m_{s_5}-1$ | $m_{s_6}$ | $m_{s_1}$ | $m_{s_2}$ | $m_{s_3}$ | $m_{s_4}$ | $m_{s_5}$ | $m_{s_6}$ | $\lambda(m_{s_4}+1)$ |
| $m_{s_1}+1$ | $m_{s_2}$ | $m_{s_3}$ | $m_{s_4}-1$ | $m_{s_5}$ | $m_{s_6}$ | $m_{s_1}$ | $m_{s_2}$ | $m_{s_3}$ | $m_{s_4}$ | $m_{s_5}$ | $m_{s_6}$ | $\lambda(m_{s_3}+m_{s_5}+m_{s_6})$ |
| $0$ | $m_{s_2}+1$ | $m_{s_3}$ | $m_{s_4}$ | $m_{s_5}-1$ | $m_{s_6}$ | $0$ | $m_{s_2}$ | $m_{s_3}$ | $m_{s_4}$ | $m_{s_5}$ | $m_{s_6}$ | $\lambda(m_{s_3}+m_{s_5}-1+m_{s_6})$ |
| $0$ | $0$ | $m_{s_3}$ | $m_{s_4}+1$ | $m_{s_5}$ | $m_{s_6}-1$ | $0$ | $0$ | $m_{s_3}$ | $m_{s_4}$ | $m_{s_5}$ | $m_{s_6}$ | $\lambda(m_{s_3}+m_{s_5}+m_{s_6}-1)$ |

### 4.3 Share-per-node: One fiber per port ($F = 1$), $M$ wavelengths and $C$ converters ($C \le 2 * M$) shared between two ports $N = 2$

In this case we proceed exactly the same as in the previous section, but now the CTMC is the 4-tuple $(m_1, r_1, m_2, r_2)$ where $m_i, r_i$ represent the number of busy wavelengths and converters in port $i$, with $i = 1, 2$. The blocking probability is also modified to include the states in which $r_1 + r_2 = C$, with blocking probability $m_1/M$ and $m_2/M$ for port 1 and 2, respectively. The analytical expressions are not given here for brevity. The number of states for the share-per-node case can be derived using the number of states for the share-per-link case with $F = 1$ which we denote as

$$S_l(M, R)=(R+1) * (R+2)/2+(M-R) * (R+1) \quad (5)$$

Then the number of states for the share-per-node case can be written when $C \le M$ as

$$S_n(M, C) = (M+1) * S_l(M, C) + (M) * S_l(M, C-1) + \cdots + (M-C+1) * S_l(M, 0) \quad (6)$$

which is $\mathcal{O}(M^2)$ when $C \ll M$.

## 5 Approximated analytical models

In this section, an approximated analytical model will be presented for the asynchronous multi-fiber buffer-less case. Note that the Markov chains' approach presented in the previous section could be adopted, although it would have critical complexity as the number of switched channels increases. Thus, a different approach is proposed here to achieve quite good matching with lower complexity. The model is based on the Equivalent Random Theory [23,24]. The model is first introduced for the share-per-link architecture and then it is extended to the share-per-node case. With the assumption of asynchronous network and variable packet length the incoming traffic is assumed to be Poisson again (rate $\lambda$) and the packet size distribution as exponential (mean $1/\mu$). These assumptions are quite realistic as shown in previous works [15]. The total load is equally distributed toward the output channels. For a matter of clarity all the variables included in the model will now be listed and explained. The model will be described immediately after. First of all we anticipate the general expression for the packet loss probability which is:

$$P_{\text{Loss}} = P_{\text{u}} + P_{\text{tr}} \cdot \left(1 - \frac{P_{\text{u}}}{P_{\text{tr}}}\right) \cdot P_{\text{bwc}} \quad (7)$$

where

- $P_u$ is the probability of having all the output channels busy independently of the state of the converters.
- $P_{tr}$ is the probability that a packet needs a converter to be sent because its incoming wavelength is busy on the output interface.
- $P_{bwc}$ is the packet loss experienced by the converters.
- $A_0$ is the average load on incoming wavelengths.
- $A_1$ is the load on a tagged outgoing wavelength.
- $A_+$ is the portion of traffic that comes from the set of converters after conversion to the tagged outgoing wavelength.
- $A_{tr}$ is the portion of traffic directed to the converters from a single busy wavelength.
- $V_{tr}$ is the variance of traffic $A_{tr}$.
- $A_{wc}$ is the total traffic that is directed to the pool of converters.
- $z$ peakedness defined as the ratio between variance and the mean of variable $A_{tr}$.

To solve the analytical problem a tagged outgoing channel is considered. This channel is loaded with an amount of traffic $A_1$ that results in:

$$A_1 = A_0 + A_+ \tag{8}$$

that is the sum of the average input load per wavelength $A_0$ plus the traffic $A_+$ that comes from the set of converters after conversion to the tagged wavelength [25]. In any case $A_1 \leq 1$, which means that the system is not overloaded and allows temporarily traffic unbalancing on the target wavelength due to wavelength conversion. Here, differently from [25], the probability $P_u$ of having all the output channels busy independently of the state of the converters can be calculated using the Erlang B-Formula with $F \cdot M$ servers loaded with $F \cdot M \cdot A_0$ as:

$$P_u = B(F \cdot M; F \cdot M \cdot A_0) \tag{9}$$

$P_{tr}$ is the probability that a packet needs a converter to be sent because its incoming wavelength is busy. If there are wavelength channels available at the output ports the packet looks for a different wavelength and uses a converter. If there are no wavelength channels available the packet is discarded. $P_{tr}$ is calculated as the joint probability that the $F$ wavelengths (one on each fiber) of the same color of the tagged packet are busy and there is at least a wavelength free at the output stage.

$$P_{tr} = (1 - P_u) \cdot B(F; F \cdot A_1) \tag{10}$$

$A_1$ in this case is assumed Poisson and as long as $A_+$ is a small fraction of $A_0$ this assumption is quite tolerable [25]. $A_{tr}$ is the portion of traffic directed to the converters from a single wavelength and is expressed as:

$$A_{tr} = A_0 \cdot P_{tr} \cdot \left(1 - \frac{P_u}{P_{tr}}\right) \tag{11}$$

where the term $\left(1 - \frac{P_u}{P_{tr}}\right)$ takes into account the fraction of overflow traffic that does not incur in output overbooking and that is already taken into account by $P_u$. The set of converters is loaded by the overflow traffic concerning all output interfaces and is calculated as the total traffic $A_{wc}$ that is directed to the $R$ converters, easily deduced from the expression 11 of $A_{tr}$:

$$A_{wc} = M \cdot F \cdot A_{tr} \tag{12}$$

The traffic $A_{wc}$ is not exponential [18] and has been characterized by the Equivalent Random Theory [23,24]. This theory allows to use the Erlang B-Formula for non-Poisson traffic streams if they are normalized to the peakedness $z$. This parameter is calculated as the ratio between the variance and the mean value of $A_{tr}$ (see formula (11) and (14)). It is an index of the variability of the traffic with comparison with the Poisson distribution for which it results $z = 1$. The 'peaky' traffic that loads the converters has a greater variability than Poisson traffic and so $z > 1$. The variance of the traffic $A_{tr}$ is evaluated through the formula [23]:

$$V_{tr} = A_{tr} \cdot \left(1 - A_{tr} + \frac{F \cdot M \cdot A_1}{F \cdot M - F \cdot M \cdot A_1 + A_{tr} \cdot F \cdot M + 1}\right) \tag{13}$$

taken from the Equivalent Random Theory and applied to the multi-fiber scheme. The peakedness $z$ can be then expressed as:

$$z = \frac{V_{tr}}{A_{tr}} \tag{14}$$

The packet loss probability $P_{bwc}$ experienced by the converters can be then expressed as [24]:

$$P_{bwc} = B\left(\frac{R}{z}; \frac{A_{wc}}{z}\right) \tag{15}$$

By using (15) the expression of $A_+$ is obtained as:

$$A_+ = A_{tr}(1 - P_{bwc}) \tag{16}$$

Finally the overall packet loss probability is formulated as:

$$P_{Loss} = P_u + P_{tr} \cdot \left(1 - \frac{P_u}{P_{tr}}\right) \cdot P_{bwc} \tag{17}$$

where, again, $\left(1 - \frac{P_u}{P_{tr}}\right)$ takes into account that part of traffic that does not occur in output contention. Previous equations can be numerically solved to determine $P_u$, $P_{tr}$, $P_{bwc}$ and calculate $P_{Loss}$ through 17.

The extension to the share-per-node case is quite straightforward. The same approach is indeed adopted. The only changes affect the expression of the variance of the traffic

$A_{tr}$ and of the total traffic $A_{wc}$ directed to the converters that become:

$$A_{wc}^{node} = M \cdot F \cdot N \cdot A_{tr} \tag{18}$$

and

$$V_{tr}^{node} = A_{tr} \times \left(1 - A_{tr} + \frac{F \cdot M \cdot N \cdot A_1}{F \cdot M \cdot N - F \cdot M \cdot N \cdot A_1 + A_{tr} \cdot F \cdot M \cdot N + 1}\right) \tag{19}$$

being the pool of converters in this case shared among all $N \times F \times M$ channels. The validation of the model through comparison with simulation results will be shown in the results section.

## 6 Numerical results

In this section, numerical results are presented with the aim to validate the analytical models and show the effectiveness of the approximated approach.

### 6.1 Exact model validation (by comparison with simulation)

In Fig. 3 results obtained by the exact model of the share-per-node configuration are validated against simulation in case of balanced traffic. Simulation results are obtained with confidence interval at 95% less than or equal to the 5% of the mean. Perfect agreement is shown for different values of load. In Figs. 4 and 5 the validation is obtained for unbalanced traffic and perfect agreement has been found again. In Fig. 6 the

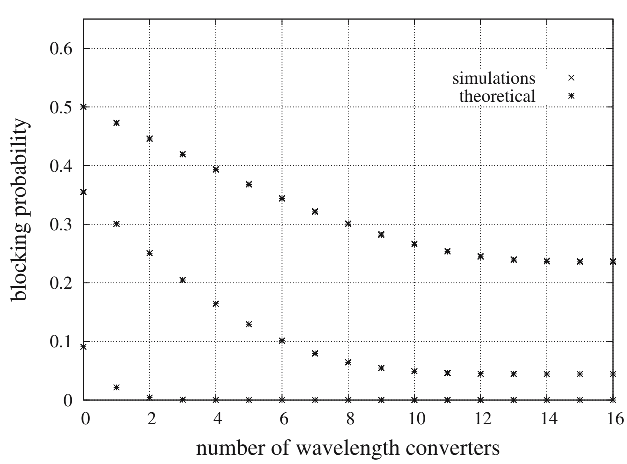


**Fig. 3** Blocking probability for share-per-node switch with $N = 2$, $M = 8$, $F = 1$ as a function of the number of shared wavelength converters varying the load $A_0 = 0.1, 0.45, 1.0$ per wavelength: comparison between exact model and simulation

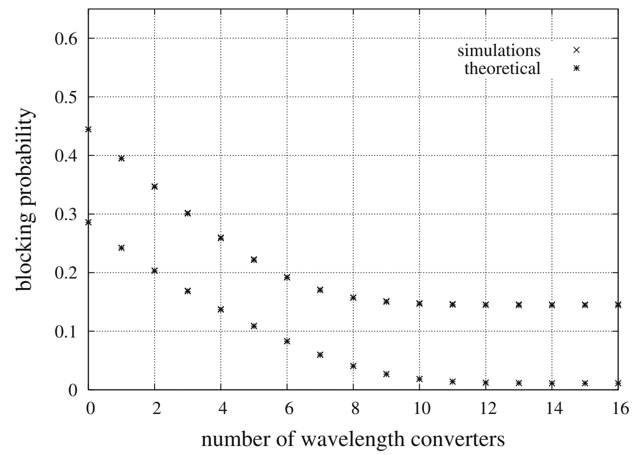


**Fig. 4** Blocking probability for share-per-node switch with $N = 2$, $M = 8$, $F = 1$ as a function of the number of shared wavelength converters for load $A_{01} = 0.8$ and $A_{02} = 0.4$ per wavelength: comparison between exact model and simulation

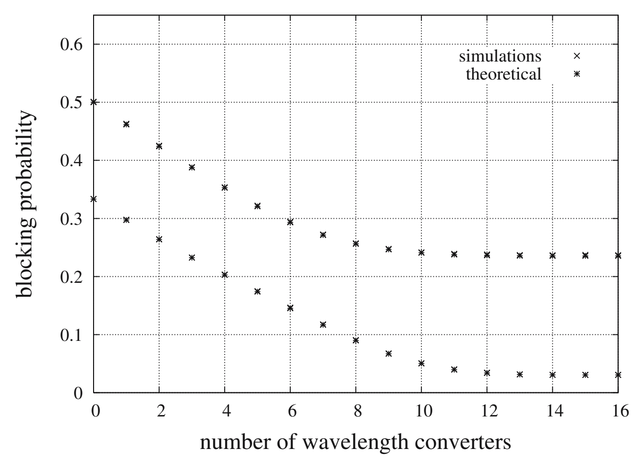


**Fig. 5** Blocking probability for share-per-node switch with $N = 2$, $M = 8$, $F = 1$ as a function of the number of shared wavelength converters for load $A_{01} = 1.0$ and $A_{02} = 0.5$ per wavelength: comparison between exact model and simulation

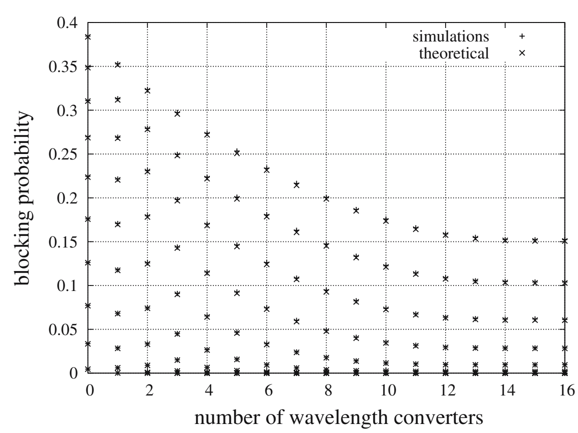


**Fig. 6** Blocking probability for multi-fiber share-per-link switch with $N = 1$, $M = 8$, $F = 2$ as a function of the number of shared wavelength converters varying the load $A_0$ from 0.05 to 0.95 per wavelength: comparison between exact model and simulation

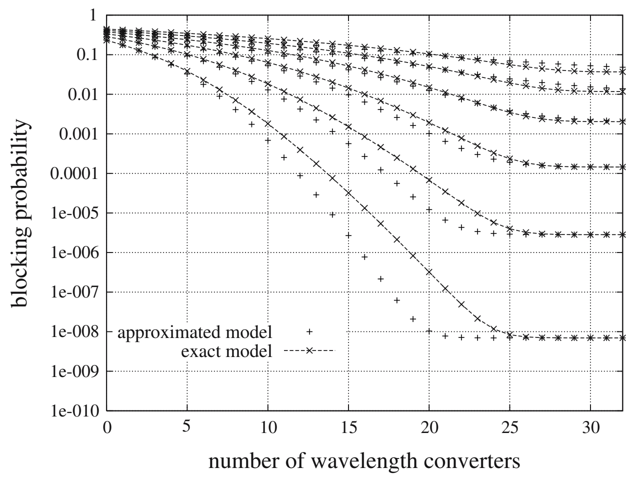


**Fig. 7** Blocking probability for share-per-link switch with $N = 1$, $M = 32$, $F = 1$ as a function of the number of shared wavelength converters varying the load $A_0$ from 0.3 to 0.8 per wavelength: comparison between exact and approximated models

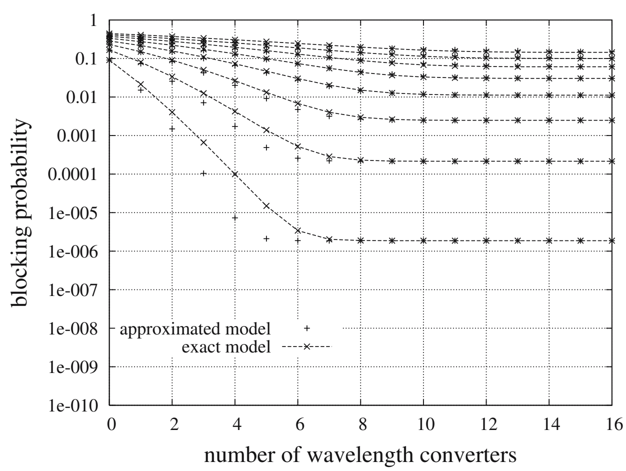


**Fig. 8** Blocking probability for share-per-node switch with $N = 2$, $M = 8$, $F = 1$ as a function of the number of shared wavelength converters varying the load $A_0$ from 0.2 to 0.8 per wavelength: comparison between exact and approximated models

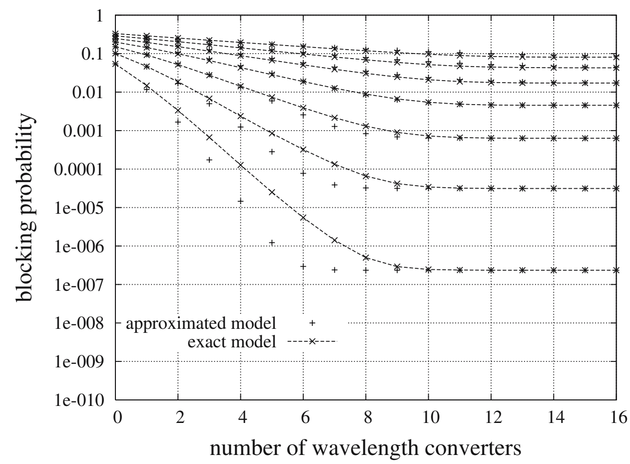


**Fig. 9** Blocking probability for share-per-link switch with $N = 1$, $M = 8$, $F = 2$ as a function of the number of shared wavelength converters varying the load $A_0$ from 0.2 to 0.8 per wavelength: comparison between exact and approximated models

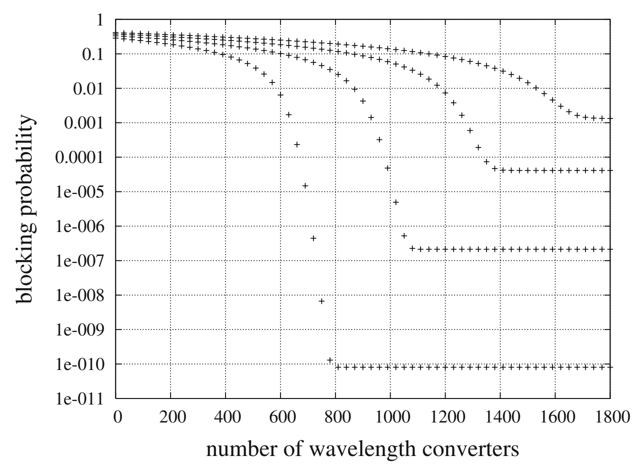


**Fig. 10** Blocking probability for share-per-node switch with $N = 64$, $M = 64$, $F = 1$ as a function of the number of shared wavelength converters varying the load $A_0$ from 0.4 to 0.7 per wavelength: approximated model

multi-fiber case is considered for $F = 2$. Also in this case the Markov chain-based approach is shown to give exact results.

On the basis of these first results the effectiveness of the Markov approach can be assessed only for very limited values of switch parameters. This analysis could be extended also to other values of $N$ and $F$ but they would be of little use as the number of states grows dramatically.

### 6.2 Approximated model evaluation (by comparison with exact model)

In Fig. 7 the exact and approximated models are compared for share-per-link switch varying the load from 0.3 to 0.8. It can be seen an underestimation of loss which tends to become slighter as the traffic increases. At medium load the estimation of loss is within the same range of loss probability. The same comment can be applied to the share-per-node switch, whose performance for $F = 1$ is represented in Fig. 8, and for the share-per-link multi-fiber option with $F = 2$ represented in Fig. 9.

In Figs. 10 and 11 the approximated model is applied in ranges where neither the exact model, because of large switch size, nor the simulation, because of low blocking probability range, is suitable. In fact, performance evaluation for very large switches and very low loss probabilities are reported for load values $A_0 = 0.4, 0.5, 0.6, 0.7$. In these figures the asymptotic values are exact, and a quite good evaluation of the number of wavelength converters needed to obtain a given probability of blocking can be obtained, although optimistic.

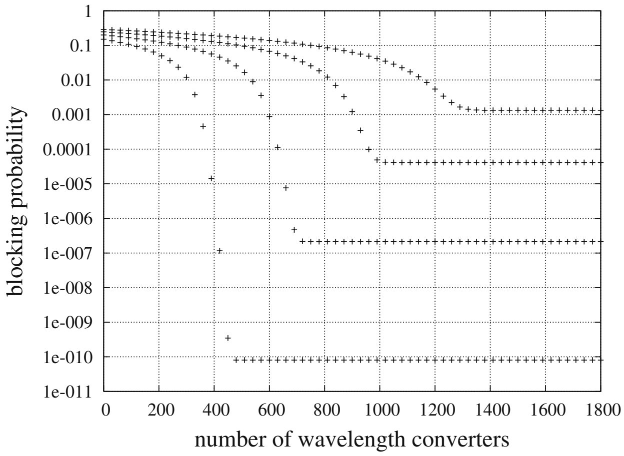


**Fig. 11** Blocking probability for share-per-node switch with $N = 64$, $M = 32$, $F = 2$ as a function of the number of shared wavelength converters varying the load $A_0$ from 0.4 to 0.7 per wavelength: approximated model

## 7 Conclusions

Blocking analysis of asynchronous optical burst switches equipped with shared wavelength converters has been performed based on three different approaches: continuous time Markov chain model, equivalent random theory model, and simulation. The first approach has been shown to provide exact results by comparison with simulation but only for very limited values of system parameters, i.e., the number of fibers and the switch size, due to the model complexity. The equivalent random theory approach gives approximated results, based mainly on the assumption of independence between loss events on output channels and on the wavelength converters' pool. In any case this approach is computationally fast and quite accurate unless traffic is very low. The asymptotic value is captured very accurately. On the other hand it is well known that simulation is not fair for very low values of loss probability. The application of the approximated approach to switch configurations and operating conditions which are untractable both with the exact model and simulation are finally shown to enforce the validity of the approach.

**Acknowledgements** The authors would like to thank the Commission of European Union for funding this work through the IST-FP6 Network of Excellence *e-Photon/One+*.

## References

[1] Xu, J., Perros, H., Rouskas, G.: Techniques for optical packet switching and optical burst switching. IEEE Commun. Mag. **39**(1), 136–142 (2001)
[2] Qiao, C., Yoo, M.: Choices features and issues in optical burst switching. Opt. Netw. Mag. **1**(2), 36–44 (2000)
[3] Turner, J.: Terabit burst switching. J. High Speed Netw. **8**(1), 3–16 (1999)
[4] Chen, Y., Qiao, C., Yu, X.: Optical burst switching: a new area in optical networking research. IEEE Netw. **18**(3), 16–23 (2004)
[5] Sriram, K., Griffith, D.W., Lee, S., Golmie, N.T.: Optical burst switching: benefits and challenge. In: Proceedings of the First International Workshop on Optical Burst Switching (WOBS 2003), Dallas/TX, October 2003
[6] Yoo, M., Qiao, C., Dixit, S.: Optical burst switching for service differentiation in the next generation optical internet. IEEE Commun. Mag. **39**, 98–104 (2001)
[7] Stavdas, A.: Architectures, technology and strategies for a gracefully evolving optical packet switching networks. SPIE Opt. Netw. Mag. **4**(3), 92–107 (2003)
[8] Maier, G., Pattavina, A.: Generalized space equivalent analysis of optical cross-connect architectures. IEEE Infocom **1**, 159–168 (2001)
[9] Eramo, V., Listanti, M., Pacifici, G.: A comparison study on the number of wavelength converters needed in synchronous and asynchronous all-optical switching architectures. IEEE/OSA J. Lightwave Technol. **21**(2), 340–355 (2003)
[10] Eramo, V., Listanti, M., Spaziani, M.: Resource sharing in optical packet switches with limited range wavelength converters. IEEE/OSA J. Lightwave Technol. **23**(2), 671–686 (2005)
[11] Norte, D., Willner, E.: All-optical data format conversions and reconversions between the wavelength and time domains for dynamically econfigurable WDM networks. J. Lightwave Technol. **14**(6), 1170–1182 (1996)
[12] Cao, X., Li, J., Chen, Y., Qiao, C.: Assembling TCP/IP packets in optical burst switched networks. In: Proc. of IEEE Globecom November 2002, pp. 2808–2812. Taipei, Taiwan
[13] Teng, J., Rouskas, G.N.: A comparison of the JIT, JET, and horizon wavelength reservation schemes on a single OBS node. In: Proceedings of the First International Workshop on Optical Burst Switching (WOBS 2003), Dallas/TX (October 2003)
[14] Yu, X., Li, J., Cao, X., Chen, Y., Qiao, C.: Traffic statistics and performance evaluation in optical burst switched networks. J. Lightwave Technol. **22**(12), 2722–2738 (2004)
[15] Izal, M., Aracil, J.: On the influence of self-similarity on optical burst switching traffic. In: Proc. of IEEE Globecom November 2002, vol. 3, pp. 2308–2312. Taipei, Taiwan
[16] de Vega Rodrigo, M., Götz, J.: An analytical study of optical burst switching aggregation strategies. The Third International Workshop on Optical Burst Switching, San Jose, Oct. 25 (2004)
[17] Laevens, K.: Traffic characteristics inside optical burst switched networks. In: Proceedings of SPIE Optical Networking and Communications (Opticomm 2002), Boston, July 29–Aug. 2, 2002, pp. 137–148
[18] Muretto, G., Raffaelli, C.: Performance evaluation of asynchronous multi-fibre optical packet switches. In: Proceeding of Optical Networks Design and Modeling (ONDM) 2006, Copenhagen, Denmark, 22–24 May 2006
[19] Danielsen, S.L., Hansen, P.B., Stubkjaer, K.E.: Wavelength conversion in optical packet switching. J. Lightwave Technol. **16**(12), 2095–2108 (1998)
[20] Li, L., Somani, A.K.: A new analytical model for multifiber WDM networks. IEEE J. Sel. Areas Commun. **18**(10), 2138–2145 (2000)
[21] Akar, N., Karasan, E., Dogan, K.: Wavelength converter sharing in asynchronous optical packet/burst switching: an exact blocking analysis for Markovian arrivals. IEEE J. Sel. Areas Commun. **24**(12, Part Supplement), 69–80 (2006)
[22] Muretto, G., Raffaelli, C.: Contention resolution in multi-fibre optical packet switches. COST291/GBOU ONNA Workshop on Design of Next Generation Optical Networks: from the Physical up to the Network Level Perspective, February 6, 2006, Gent, Belgium

[23] Wilkinson, R.I.: Theories for toll traffic engineering in the USA. Bell Syst. Tech. J. **35**(2), 569–582 (2004)
[24] Fredericks, A.A.: Congestion in blocking system. A simple approximation technique. Bell Syst. Tech. J. **59**, 805–826 (1980)
[25] Casoni, M., Merani M.L.: On the performance of TCP over optical burst switched networks with different QoS classes. In: Proceedings QoSIP 2005, pp. 574-585. Catania, Italy, 2–4 February 2005

## Author Biographies

**Pedro Reviriego** received the M.Sc. and Ph.D. degrees (Honors) from Technical University of Madrid in 1994 and 1997, both in Telecommunications Engineering. From 1997 to 2000 he was an R&D engineer at Teldat working on router implementation. In 2000 he joined Massana to work on the development of 1000BaseT transceivers. During 2003 he was a Visiting professor at University Carlos III. From 2004 to 2007 he was Distinguished Member of Technical Staff with LSI Corporation working on the development of Ethernet transceivers. He is currently with Universidad Antonio de Nebrija. His research interests are fault tolerant systems, performance evaluation of communication networks and the design of physical layer communications devices. He has authored numerous papers in international conferences and journals. He has also participated in the IEEE 802.3 standardization for 10GBaseT.

**Anna Maria Guidotti** is a Ph.D. student at the University of Bologna. She received the M.S. degree in Telecommunications Engineering from the University of Bologna, Italy, in 2007. She is currently working in optical networks, focusing her research on optical access and optical packet/burst switching. Anna M. Guidotti was involved in the European project e-Photon/One.

**Carla Raffaelli** is an associate professor in switching systems and telecommunication networks at the University of Bologna. She received the M.S. and the Ph.D. degrees in electrical engineering and computer science from the University of Bologna, Italy, in 1985 and 1990, respectively. Since 1985 she has been with the Department of Electronics, Computer Science and Systems of the University of Bologna, Italy. Her research interests include performance analysis of telecommunication networks, switching architectures, protocols and broadband communication. Since 1993 she participated in European funded projects on optical packet-switched networks, the RACE-ATMOS, the ACTS-KEOPS the IST-DAVID, e-photon/One projects. She is now active in the EU funded BONE network of excellence. She also participated in many national research projects on telecommunication networks. She is the author of many technical papers on broadband switching and network modeling and acts as a reviewer for top international conferences and journals. She is author or co-author of more than 100 conference and journal papers mainly in the field of optical networking and performance evaluation.

**Javier Aracil** received the M.Sc. and Ph.D. degrees (Honors) from Technical University of Madrid in 1993 and 1995, both in Telecommunications Engineering. In 1995 he was awarded with a Fulbright scholarship and was appointed as a Postdoctoral Researcher of the Department of Electrical Engineering and Computer Sciences, University of California, Berkeley. In 1998 he was a research scholar at the Center for Advanced Telecommunications, Systems and Services of The University of Texas at Dallas. He has been an associate professor for University of Cantabria and Public University of Navarra and he is currently a full professor at Universidad Autónoma de Madrid, Madrid, Spain. His research interests are in optical networks and performance evaluation of communication networks. He has authored more than 75 papers in international conferences and journals.