Analysis of the Processing and Sojourn Times of Burst Control Packets in Optical Burst Switches

Luis de Pedro, Javier Aracil, José Alberto Hernández and José Luis García-Dorado
Networking Research Group, Depto. Ingeniería Informática
Universidad Autónoma de Madrid, Spain

***Abstract*—In Optical Burst Switched networks, when a Burst Control Packet arrives at a core node, its scheduling algorithm looks up its internal reservation table for an available gap at which to accomodate the incoming burst. The processing time of the BCP is variable, and depends on the number of available gaps over which to perform the search.**

This work analyses the gap distribution in such reservation table, and derives an expression for the waiting time of BCPs, assuming LAUC-VF scheduling. Our findings can be applied to the dimensioning of the offset time values in OBS networks, for a realistic case of BCP variable processing times.

***Index Terms*—Optical Burst Switching, Burst Control Packet processing time, void filling scheduling algorithms.**

I. Introduction

Optical Burst Switching (OBS) [1] provides intermediate switching granularity between optical circuit and packet switching. For each data burst, a Burst Control Packet (BCP) is sent ahead in order to announce the burst arrival at intermediate OBS nodes. With the information carried by the BCP, the node's scheduler attempts to find an available gap at which to allocate the forthcoming burst. If more than one suitable gap is found, then the Switch Control Unit (SCU) chooses one of them according to some scheduling algorithm (see Horizon [2], LAUC-VF [3], Min-SV [4] and MinVoids [5]). If all output wavelengths are occupied and no available gap is found, the data burst is then dropped.

LAUC-VF is known to outperform since it considers all the possible gaps among already scheduled data bursts, that is, it employs *void filling*. Clearly, this algorithm requires to keep track of the starting times and durations of all scheduled time intervals (gaps) in a given internal structure. Upon BCP arrival, the algorithm looks up this structure for an available gap, and updates it if the burst is successfully fitted in a gap. Thus, for scheduling algorithms with void filling, *the algorithm's execution time is variable*, since the processing time spent directly depends on the set of available gaps over which to perform the search. Clearly, such number of gaps also depends on the switch's load.

This paper aims to characterise the sojourn time of BCPs at core OBS nodes. Essentially, incoming BCPs are first stored in a queue, and then served following a First-Come First-Served basis (see Fig. 1). Thus, the total sojourn time of a BCP constitutes the sum of the two components: waiting time in queue and processing time.

Under the assumption of Poissonian BCP arrivals [6], the $M/G/1$ model can be used to obtain such waiting delay in queue. The processing time accounts for the time spent until a suitable gap at which to allocate the incoming burst is found. More precisely, if the gap information is stored, for instance, as a binary tree, then the scheduling time is proportional to the logarithm (base 2) of the number of gaps over which to perform the search.

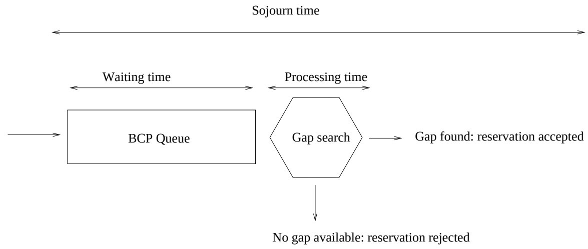

Fig. 1. SCU model

Indeed, the accurate modelling of the processing time of BCPs is key in designing the offset time given to every source-destination path in the OBS network, since BCPs' sojourn time consume offset time.

II. Preliminaries

Let us consider that the SCU features the LAUC-VF algorithm, and let the available gap information be structured as a binary tree, even though the methodology presented in this paper can be used for any other search algorithm. In what follows, a *gap* (or void) denotes the time interval between two already scheduled bursts, as shown in Fig. 2.

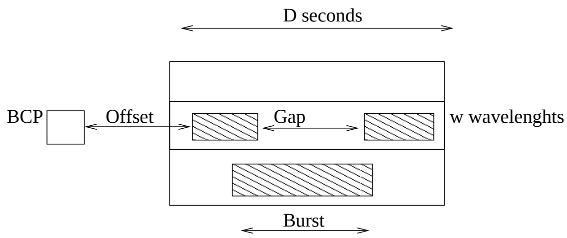

Fig. 2. Example of burst scheduling and resulting gaps

Essentially, when a BCP arrives at a core node with w wavelengths, a time-horizon of D units of time is defined. This time-horizon can be set, for instance, to the maximum offset time. We note that the scheduling agent keeps track of the unscheduled time-slots (gaps) in all wavelengths. Then, upon

BCP arrival, the scheduling agent runs a search algorithm, that outputs *a list of possible gaps* over which the burst fits. Then, the scheduling algorithm selects the most suitable gap according to the metric defined by the particular scheduling algorithm (LAUC-VF in our case).

For simplicity, we shall consider that the time taken to perform the optimal choice among the available gaps (scheduling time) is very low compared to the time spent in searching for available gaps (searching time). Processing and searching time are thus considered the same amount.

III. Analysis of gap searching time

Clearly, the gap searching time, depends on the probability distribution of the number of gaps over which to perform the search. In this light, the more bursts scheduled, the more gaps between them, hence the searching time increases with the number of successfully scheduled bursts. The following derives an exact equation for modelling the gap searching time.

A. Burst arrival process and distribution of number of gaps

Let BCP arrivals follow a Poisson process with rate λ_{BCP}. The burst arrival process must take into account both the BCP arrival process and the offset-time values, which shall be assumed uniformly distributed. The following lemma states a fundamental result about the burst arrival process.

Lemma 1 If the BCP arrival process is Poisson, then the burst arrival process is Poisson, regardless of the offset-time distribution.

Proof: Let $\{Y(t), t > 0\}$ be the BCP arrival process (Poisson with rate λ), and let $\{Z(t), t > 0\}$ be the burst arrival process. Also, let X be the offset time with probability density function $f_X(x), x > 0$. Then,

$$\begin{aligned}&\mathbb{P}(Z(t_0+\Delta t) - Z(t_0) = 1) = \\ &= \int_0^\infty \mathbb{P}(Y(t_0 - x + \Delta t) - Y(t_0 - x) = 1 | X = x) \\ &f_X(x)dx = \\ &= \int_0^\infty (\lambda \Delta t + o(\Delta t)) f_X(x) dx = \lambda \Delta t + o(\Delta t)\end{aligned}$$

and the burst arrival process is Poisson. □

Furthermore, the number of accepted data bursts is thus Poisson (as seen above) with rate $\lambda = (1 - \mathbb{P}_b)\lambda_{BCP}$, where $\mathbb{P}_b$ denotes the burst blocking probability.

Now, the following observation is key for deriving the distribution of number of gaps: *Each accepted burst generates an extra gap*. Basically, when a data burst is successfully scheduled in a gap, such gap is effectively split into two, thus creating two new gaps. Thus, the gap arrival process and the accepted burst arrival process are the same.

Consequently, let $N_g^{(ss)}$ be the number of gaps under steady-state conditions, and recall that the scheduling horizon time is equal to D units of time. Then the probability density function (pdf) of gaps under steady-state conditions is given by:

$$P(N_g^{(ss)} = n) = \frac{(\lambda D)^{(n-1)}}{(n-1)!} e^{-\lambda D}, \quad n = 1, 2, \dots \qquad (1)$$

B. Gap distribution at BCP arrival

Recall that eq. 1 gives the pdf of gaps in steady-state conditions. The following derives the gap distribution *at BCP arrivals*.

Let $t = 0$ denote the arrival time of a given BCP ("tagged" BCP, see fig. 3) with maximum offset time D, thus attempting to make a reservation somewhere in $[0, D]$. The next step is to derive the number of successfully scheduled bursts in the horizon time of such tagged BCP. Clearly, BCP arrivals at time $t < -D$ do not count since they could never possibly schedule their associated bursts within time $t \in [0, D]$, and so applies to BCP arrivals after $t > 0$ for the tagged BCP would have preference over them since it arrived earlier. Thus, the number of BCPs that may attempt for burst reservation in the horizon time of the tagged BCP constitutes only a portion of the arrivals in $[-D, 0]$. Clearly, some of such arrivals in $[-D, 0]$ attempt for reservation in $[-D, 0]$, and some others attempt over the time interval $[0, D]$. The question is how many of them attempt for reservation over $[0, D]$.

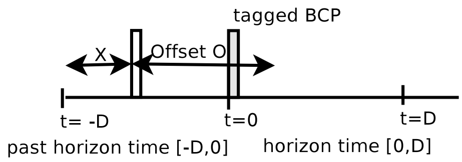

Fig. 3. Tagged BCP arrival

To do so, let $O \sim U(0, D)$ denote the offset time of a BCP which arrives in the "past-horizon" time (that is, in $[-D, 0]$) of the tagged BCP, and let X refer to its arrival time. Clearly, since BCPs arrive following a Poisson process, X is uniformly distributed over $[0, D]$. Then, the probability to actually attempt for a reservation over the horizon time of the tagged BCP is:

$$P = \int_0^D P(O > D - x) f_X(x) dx = \int_0^D \frac{D-x}{D}\frac{1}{D} dx = \frac{1}{2} \qquad (2)$$

Thus, only half of the arrivals in $[-D, 0]$ actually attempt for a reservation in the horizon time of the tagged BCP. Hence, *at BCP arrivals*, the distribution of the number of gaps in the horizon time of the tagged BCP, $N_g^{(BCP)}$, is given by:

$$P(N_g^{(BCP)} = n) = \frac{(\lambda_g D)^{(n-1)}}{(n-1)!} e^{-\lambda_g D}, \quad n = 1, 2, \dots \qquad (3)$$

where $\lambda_g = \frac{1}{2}(1 - \mathbb{P}_b)\lambda_{\text{BCP}}$.

C. Gap searching time

Finally, let T_s denote the SCU service time. Then, assuming a binary search algorithm is employed, the searching time is proportional to the logarithm (base 2) of the number of gaps over which the search is performed. That is:

$$T_s = k \log(N_g^{(BCP)}) \tag{4}$$

where $k > 0$ is an arbitrary constant that denotes the cost per search (in units of time). Therefore:

$$\mathbb{P}(T_s = t_n) = P(N_g^{(BCP)} = n), \quad n = 1, \ldots \tag{5}$$

where $t_n = k \log(n)$, $n = 1, 2, \ldots$ are discrete values. The service time distribution function $F_{T_s}(t) = \mathbb{P}(T_s < t)$, which is necessary for the next section, is thus given by:

$$F_{T_s}(t) = \sum_{i=1}^{n} \frac{(\lambda_g D)^{(i-1)}}{(i-1)!} e^{-\lambda_g D} = \frac{\Gamma_{\text{inc}}(\lambda_g D, n)}{\Gamma(n)}, \quad t \geq 0 \tag{6}$$

where n satisfies $t_n \leq t < t_{n+1}$, and the incomplete gamma function is $\Gamma_{\text{inc}}(x, n) = \int_x^\infty t^{n-1} e^{-t} dt$.

IV. Waiting time distribution

To analyse the waiting time pdf in the M/G/1 queue we follow [7, chapter 5], which gives:

$$f_{W_q}(t) = (1-\rho) \sum_{n=0}^{\infty} \rho^n f_R^{(n)}(t), \quad t \geq 0 \tag{7}$$

where ρ is the M/G/1 utilisation factor and $R^{(n)}(t)$ is the n-th convolution of the service time residual life pdf $f_R(t)$ [7, equation 5.38]:

$$f_R(t) = \frac{1}{Et}(1 - F_{T_s}(t)), \quad t \geq 0 \tag{8}$$

where Et is the mean service time in the SCU.

Using eq. 6 and 8, $f_R(t)$ arises easily as:

$$f_R(t) = \frac{1}{Et}\left(1 - \frac{\Gamma_{\text{inc}}(\lambda_g D, n)}{\Gamma(n)}\right), \quad t \geq 0 \tag{9}$$

The infinite sum given by eq. 7 can be truncated to L terms since the values for large n decreases rapidly with ρ^n, $\rho \leq 1$. Thus:

$$f_{W_q}(t) \approx (1-\rho) \sum_{n=0}^{L} \rho^n f_R^{(n)}(t), \quad t \geq 0 \tag{10}$$

V. Simulation results

Fig. 4 shows the waiting time pdf of BCPs arriving at a core OBS node with obtained both theoretically and via simulation. The values of the simulation parameters are: BCP arrival rate $\lambda_{BCP} = 5.0$ bursts per unit of time, maximum offset-time $D = 10$ units of time, cost per search $k = 0.01$ units of time, fixed burst length $c = 5.0$ units of time and $w = 4$ wavelengths. The blocking probability has been considered close to the Erlang-B, since this is the case for OBS switches operating at low loads [8].

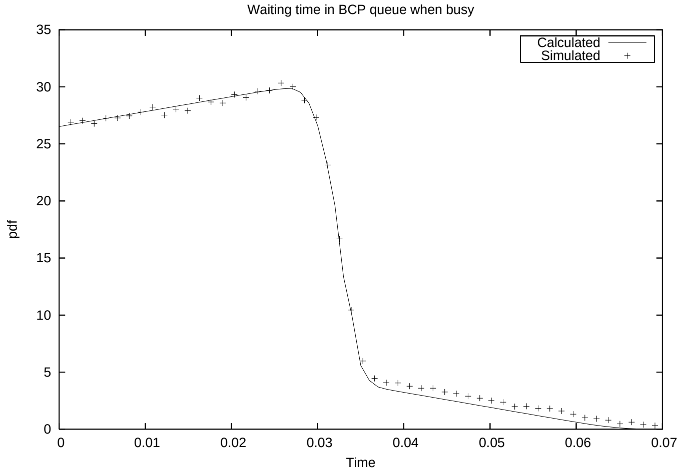

Fig. 4. Waiting Time for a BCP in the queue. Offset: Uniform=10.0, Constant burst length = 5.0, k = 0.01, BCP arrival rate = 5.0

A number of $L = 3$ terms has been considered to estimate $f_{W_q}(t)$ as given in eq. 10, since adding terms after this value does not improve the accuracy of f_{W_q}. As shown, the theoretical results accurately match the simulation values.

VI. Summary and conclusions

This work provides an accurate model for the processing time of BCPs at core OBS nodes. The model shows that: (1) such value is variable, (2) follows a Poisson distribution if the BCPs arrive on a Poisson process, (3) depends on the number of searchs (or gaps) required by the scheduling algorithm, thus on the switch's load, characterised by: λ and P_b, and (4) can be applied to a M/G/1 queue to obtain the BCP waiting time distribution.

This result is key in dimensioning the offset time values between BCPs and data bursts.

Acknowledgements

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