Accelerating SVM training: beyond SMO

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Support Vector Machines

- Standard, robust method for classification.
- Extensions for regression and novelty detection.
- Very fast algorithms available (PEGASOS, LIBLINEAR) for the linear case.
- For the non-linear case, a dual optimization problem is solved.

SVM optimization problem

$$\begin{array}{ll} \min_x & \frac{1}{2}x^T K x - x \cdot p \\ \text{s.t} & \begin{cases} 0 \leq x \leq C \\ x \cdot y = \Delta \end{cases} \end{array}$$

- Very simple problem: quadratic objective and linear constraints.
- Standard, well understood algorithms from optimization theory available: Inner Point methods, Projected Newton, etc.
- However...

Problems

- Methods with fast convergence require using Hessian or inverse of Hessian information.
- Hessian K size of the dataset and non-sparse, K^{-1} costly to compute ($O(N^3)$).
- Prohibitive for medium-sized problems.

$$\min_x \quad \frac{1}{2}x^T K x - x \cdot p$$
$$\text{s.t} \quad \begin{cases} 0 \leq x \leq C \\ x \cdot y = \Delta \end{cases}$$

Sequential Minimal Optimization

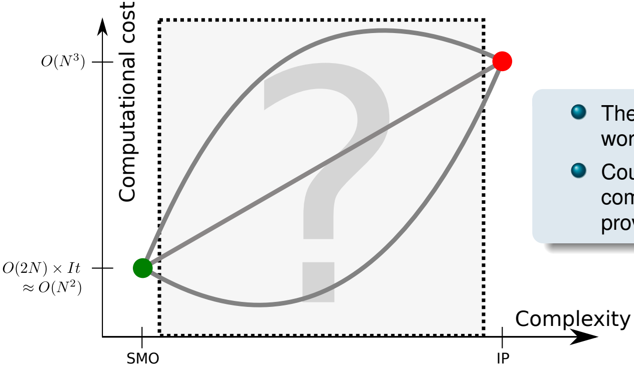
- State of the art algorithm, implemented in LIBSVM.
- At each iteration, update only the two “most violating” entries of x.
- Large number of iterations, but each of them at linear cost.
- Only 2 rows of K are used at each iteration: allows for K larger than memory.

The SMO algorithm

1. $x \leftarrow 0$, compute $\nabla f(x)$.
2. Find aprox. “best” updating direction d with 2 non-zero entries $(O(2N))$.
3. Compute optimal stepsize δ $(O(1))$.
4. Update $x' = x + \delta d$ $(O(2))$.
5. Update gradient $\nabla f(x)$ $(O(2N))$.
6. Back to 2 until convergence.

- Step 2 can be (roughly) done by looking for the largest entries of $\nabla f(x)$ taking restrictions into account, and selecting the two best ones.

The big picture



- The more complex the worse?
- Could intermediate complexity algorithms provide better results?

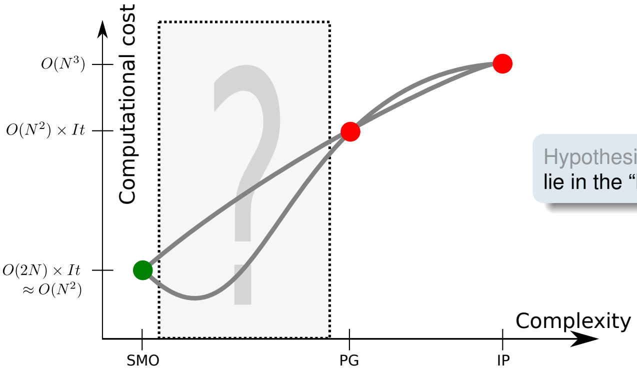
Projected Gradient

- IP is costly because it requires full Hessian information.
- SMO is very simple because it only uses 2 entries of the gradient.
- Natural intermediate algorithm: **projected gradient**.

1. $x \leftarrow 0$, compute $\nabla f(x)$.
2. Compute optimal stepsize δ ($O(N^2)$).
3. Update and project back: $x' = [x + \delta d]_P$ ($O(N)$).
4. Update gradient $\nabla f(x)$ ($O(N^2)$).
5. Back to 2 until convergence.

- Steps 2 and 4 involve a cost $O(NM)$, M number of non-zero components in d.
- Moral: sparsity in the updating direction is desirable. PG non-sparse.

The big picture revisited



Hypothesis: improvements should lie in the “low-complexity” area.

We should...

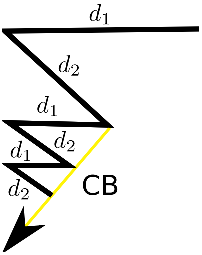
- ... avoid using the full Hessian.
- ... generate sparse updating directions d.
- ... find a balance between sparsity and usefulness of d.

Two algorithms proposed

Cycle-Breaking $\longrightarrow$ Momentum SMO

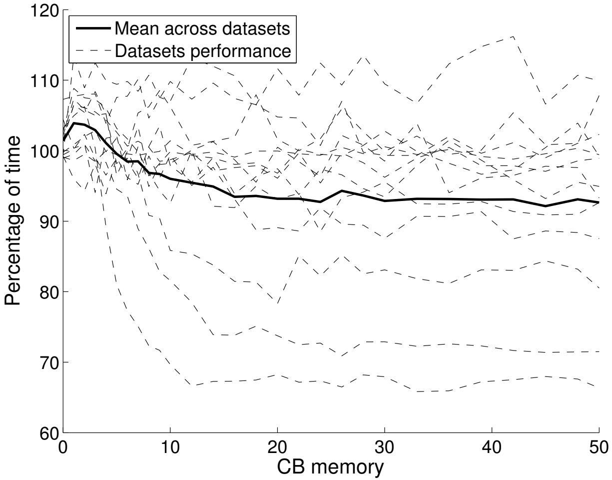
Cycle-Breaking

- "Zigzagging" is common in SMO.
- A sequence of updating directions $d_1, d_2, \ldots, d_M$ appears repeatedly during the run of the algorithm $\longrightarrow$ Cycles.
- If after doing updates along $d_1, d_2, \ldots, d_M$, SMO selects d_1 again for update, it might well happen that afterwards we will have again $d_2, \ldots, d_M$.



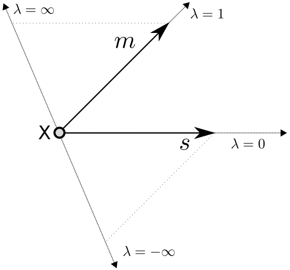
- Keep track of the τ last updating directions in a queue.
- If current updating direction is present in the queue, suppose a cycle is going on.
- Update following the direction of the cycle v (sum of previous updates).
- Sparsity is guaranteed through τ.
- Cost of a cycle-breaking update: $O(N \times \tau^2)$.

Cycle-Breaking results



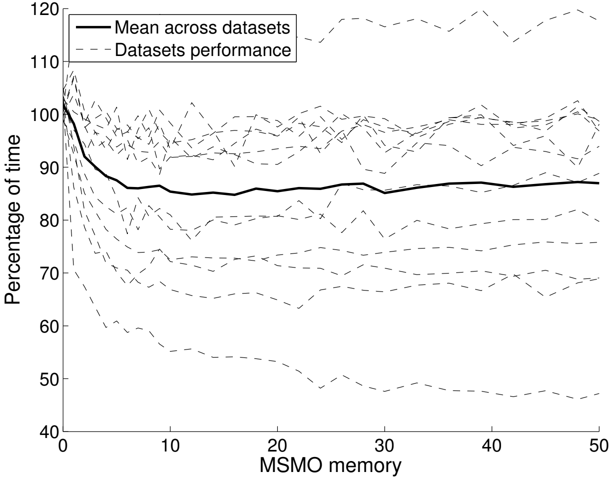
Momentum SMO

- Neural Networks: a momentum term helps to capture the "general direction" of movement.
- Classic momentum: $d_t = (1 - \lambda_t)s_t + \lambda_t m_t$, s_t SMO update, $m_t = x_t - x_{t-1}$.
- m_t non-sparse for t large.
- Limited momentum: only τ past updates, $m_t = \sum_{r=t-\tau}^{t-1}(1 - \lambda_t)\delta_t s_t$.

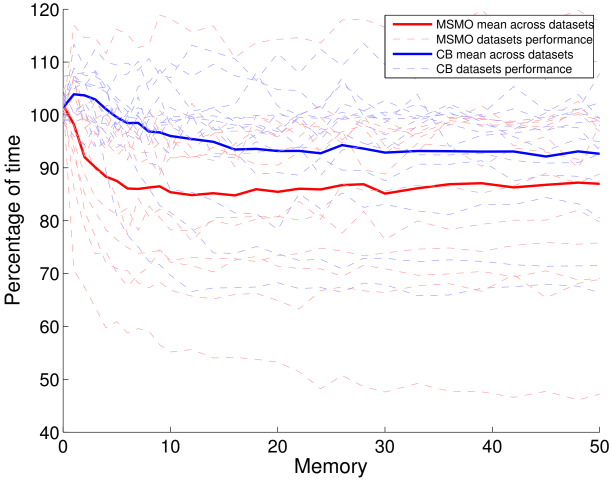


- Update as $x_{t+1} = x_t + \delta((1 - \lambda_t)s_t + \lambda_t m_t)$.
- Both the tradeoff parameter λ_t and the updating step δ_t computed in closed form.
- Optimization along a 2D halfspace.
- By storing calculations from τ previous iterations, cost $\approx O(5N)$ per iteration.

Momentum SMO results

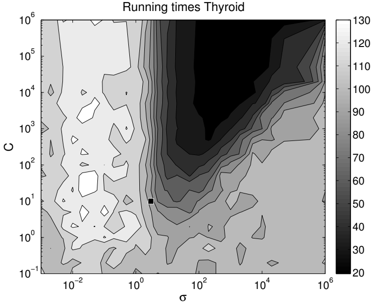


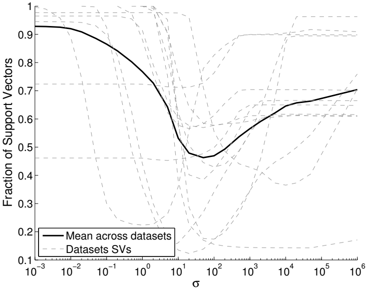
Methods comparison



Why the methods behave better in some datasets?

$\longrightarrow$ Structure of the kernel space

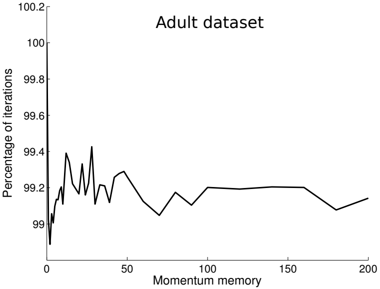


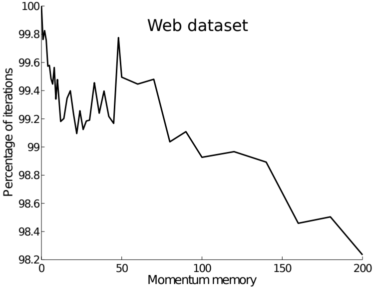


- Large C $\longrightarrow$ "unbounded problem" $\longrightarrow$ fewer hits with boundaries.
- Intermediate σ $\longrightarrow$ less SV $\longrightarrow$ smaller effective dimension.

Drawbacks

- The savings are not large enough to overthrow standard SMO.
- These methods seem to work poorly for large datasets.





Currently working on

- Adding shrinking techniques to the method $\longrightarrow$ reduce the effective dimensionality of the problem.
- For quadratic functions (like SVM) momentum with specific choices of δ, λ can be shown to be equivalent to the Conjugate Gradient method. Might be applicable here.

I will appreciate any suggestions / feedback.

Thanks for your attention

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